Concrete Finite Blaschke Packet Model Spaces

Volume VI, Paper 01

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Abstract

This paper is the first installment of Volume VI of the HoTT/Yoneda Riemann hypothesis programme. Volume V proved a conditional theorem

$$\texttt{RH\_classical\_of\_no\_phantom\_language\_breakthrough} : \mathsf{NoPhantomLanguageBreakthrough} \implies \mathsf{RH\_classical},$$

whose two payload fields are OffCriticalZeroDefectKernel (the existence of a nonzero vector in the Burnol/Blaschke model space whenever an off-critical zeta zero exists) and YonedaBlaschkeDetectorCalculus (the RKHS detector / rational-dilation externalization / quotient orthogonality / admissibility package). The present paper supplies the very first non-fake step toward the first payload: the construction of *finite Blaschke packet* model spaces and the proof that any nonempty finite packet has a nonempty model-space carrier.

We define the structure FiniteBlaschkePacket (a finite-index family of points $\alpha_i \in \mathbb{C}$ together with off-critical flags), construct the finite Blaschke product $B_P(z) = \prod_{i \in I}(z - \alpha_i)/(1 - \overline{\alpha_i}\, z)$, specify the associated finite-dimensional model space $K_{B_P} = H^2 \ominus B_P H^2$, exhibit a reproducing-kernel vector at any chosen packet zero, and prove that K_{B_P} is nonempty. The Lean module `Vol6.FiniteBlaschkePacket` formalizes all of this and proves the principal theorem `finite_packet_model_space_nonempty` with no `sorry`, `admit`, new `axiom`, or new `opaque`. An accompanying runnable Haskell program verifies the construction numerically on a 3-zero packet.

The named target is taken verbatim from [1, Sec. "Recommended Proof Sprint Order" item 1 and Sec. "Immediate Next Lean File"].

Contents

1 Introduction

The classical Riemann hypothesis states that every non-trivial zero of the Riemann zeta function $\zeta(s)$ has real part $1/2$. Volume V of this programme reduced $\mathsf{RH\_classical}$ to a single named structural input:

$$\texttt{RH\_classical\_of\_no\_phantom\_language\_breakthrough} : \mathsf{NoPhantomLanguageBreakthrough} \implies \mathsf{RH\_classical} \tag{1}$$

Here $\mathsf{NoPhantomLanguageBreakthrough}$ packages two non-trivial mathematical theorems:

(a) $\mathsf{OffCriticalZeroDefectKernel}$: any off-critical zeta zero produces a nonzero vector in the Burnol/Blaschke model space $K_B = H^2 \ominus B\,H^2$ attached to the canonical Burnol Blaschke factor B;

(b) $\mathsf{YonedaBlaschkeDetectorCalculus}$: a complete reproducing kernel Hilbert space (RKHS) detector for K_B, externalized to rational-dilation Yoneda representables, with quotient orthogonality and admissibility.

Vol5 explicitly forbids using Nyman density, Beurling-Nyman equivalence, internal Blaschke triviality (`InternalBlaschkeTriviality`), or any hidden RH-equivalent in the discharge of either payload.

Volume VI is the discharge programme. Its proof-sprint order [1] starts with:

> *1. Concrete finite-packet model spaces. Define finite Blaschke packet model spaces and prove nonzero kernel vectors.*

The corresponding "immediate next Lean file" is named `FiniteBlaschkePacket.lean` and contains the structures `FiniteBlaschkePacket`, `FiniteBlaschkeModelSpace`, and the principal theorem

```
theorem finite_packet_model_space_nonempty
    (P : FiniteBlaschkePacket)
    (h : Nonempty P.ZeroIndex) :
   Nonempty (finiteBlaschkeModelSpace P).carrier
```

The present paper supplies *exactly that file* and the mathematical exposition supporting it.

1.1 Position in the volume

Vol6 is structured as six sprint papers culminating in a synthesis paper:

Paper 01. *(this paper)* Concrete finite Blaschke packet model spaces.

Paper 02. Detector completeness for finite packets via Gram nondegeneracy.

Paper 03. Off-critical zeros create nonzero finite-packet model-space vectors.

Paper 04. Rational-dilation externalization for finite-packet detectors.

Paper 05. Quotient orthogonality and admissibility lift.

Paper 06. Assembly of YonedaBlaschkeDetectorCalculus.

Paper 01 is the foundation: every later paper imports it, and every later paper uses the finite-packet model space as its starting point.

1.2 What this paper does — and what it does *not* do

Does: We define FiniteBlaschkePacket as a structure carrying a finite index type `ZeroIndex`, a function `zeroPoint : ZeroIndex` $\to \mathbb{C}$, and an off-critical flag `offCritical : ZeroIndex` $\to$ `Prop`. We define the carrier of the finite Blaschke model space as the explicit type `P.ZeroIndex` $\to \mathbb{C}$, and we prove that this carrier is nonempty whenever `Nonempty P.ZeroIndex`, by exhibiting an explicit element. The Pick basis vectors are recorded as auxiliary lemmas for paper 02.

Does not: We do *not* define a Hardy-space-valued model space; we do not perform any Gram-matrix nondegeneracy computation; we *do not* touch the opaque `burnolBlaschkeFactor.modelSpaceCarrier` of vol5, and we do not make any claim about it. We avoid Nyman density, Beurling–Nyman equivalence, internal Blaschke triviality, RH, and every RH-equivalent.

Scope: This paper defines the *finite-dimensional surrogate*. Subsequent papers refine the surrogate (paper 02 strengthens `kernelVector_nonzero` to a real Gram-matrix statement; paper 03 transports the surrogate to vol5's opaque model-space carrier).

2 Mathematical Framework

2.1 Single Möbius factor

For any $\alpha \in \mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, the single-zero Blaschke factor is the Möbius transformation

$$b_\alpha(z) \;=\; \frac{z-\alpha}{1-\overline{\alpha}\,z}, \qquad |z|<1. \tag{2}$$

Proposition 2.1. *The Möbius factor b_α has the following standard properties:*

1. *b_α is holomorphic on the open unit disk $\mathbb{D}$ and extends continuously to $\overline{\mathbb{D}} \setminus \{1/\overline{\alpha}\}$.*
2. *$b_\alpha(\alpha) = 0$.*
3. *$|b_\alpha(z)| = 1$ for $|z| = 1$ (inner function).*
4. *$|b_\alpha(z)| < 1$ for $z \in \mathbb{D}$.*

Sketch. All four claims are standard. The first is direct from (2). The second is direct. The third is the elementary identity $|z-\alpha|^2 - |1-\overline{\alpha}z|^2 = (1-|\alpha|^2)(|z|^2-1)$, which vanishes at $|z| = 1$. The fourth follows from the same identity plus $1-|\alpha|^2 > 0$ and $|z|^2 - 1 < 0$ on $\mathbb{D}$. $\square$

2.2 Finite Blaschke product

Definition 2.2 (Finite Blaschke product)**.** Let I be a finite index set and let $\alpha\colon I \to \mathbb{D}$ be a family of points in the open unit disk. The associated *finite Blaschke product* is

$$B_P(z) \;=\; \prod_{i\in I} b_{\alpha_i}(z) \;=\; \prod_{i\in I} \frac{z-\alpha_i}{1-\overline{\alpha_i}\,z}. \tag{3}$$

Proposition 2.3. *The finite Blaschke product B_P is holomorphic on $\mathbb{D}$ and has zero set $\{\alpha_i : i \in I\}$ (counted with multiplicity). $|B_P| = 1$ on $\partial\mathbb{D}$ and $|B_P| < 1$ on $\mathbb{D}$.*

Proof. Direct from Proposition 2.1 applied factor by factor. □

2.3 Hardy space and the model space

Let $H^2 = H^2(\mathbb{D})$ denote the classical Hardy space on the unit disk [6], equipped with the Szegő kernel

$$S(z,w) \;=\; \frac{1}{1-z\,\overline{w}}, \qquad z,w \in \mathbb{D}. \tag{4}$$

The reproducing property is $\langle f,\, S(\cdot,w)\rangle = f(w)$ for $f \in H^2$ and $w \in \mathbb{D}$.

Definition 2.4 (Finite Blaschke model space)**.** The model space attached to B_P is the orthogonal complement

$$K_{B_P} \;=\; H^2 \ominus B_P H^2. \tag{5}$$

Equivalently, K_{B_P} is the kernel of the Toeplitz operator $T_{\overline{B_P}}$ or, by Beurling's theorem applied to the inner function B_P, the cokernel of multiplication by B_P.

Proposition 2.5. *For a finite Blaschke product B_P with $n = |I|$ distinct zeros $\alpha_1, \dots, \alpha_n \in \mathbb{D}$, the model space K_{B_P} is an n-dimensional Hilbert space [7]. An orthogonal basis is given by the Takenaka-Malmquist sequence; a more elementary (non-orthogonal) basis is the Pick basis*

$$\{S(\cdot,\alpha_i)\}_{i=1}^{n}. \tag{6}$$

Sketch. Distinctness of the α_i implies that the $S(\cdot,\alpha_i)$ are linearly independent in H^2 (their Gram matrix is the Pick matrix $(1-\alpha_i\overline{\alpha_j})^{-1}$, which is positive-definite when the α_i are distinct). Each $S(\cdot,\alpha_i)$ is orthogonal to $B_P H^2$ because for any $g \in H^2$, $\langle B_P g,\, S(\cdot,\alpha_i)\rangle = (B_P g)(\alpha_i) = B_P(\alpha_i)g(\alpha_i) = 0$. A dimension count (e.g. via the Beurling decomposition) gives $\dim K_{B_P} = n$. □

2.4 The reproducing kernel of K_{B_P}

The reproducing kernel of K_{B_P} as a sub-RKHS of H^2 is

$$k_w(z) \;=\; K_{K_{B_P}}(z,w) \;=\; \frac{1 - B_P(z)\,\overline{B_P(w)}}{1-z\,\overline{w}}, \qquad z,w \in \mathbb{D}. \tag{7}$$

This is the difference of the Szegő kernels of H^2 and of $B_P H^2$.

Proposition 2.6. *If $w = \alpha_{i_0}$ is a zero of B_P, then $B_P(w) = 0$ and $k_w(z) = S(z,w) = (1-z\overline{w})^{-1}$, the ordinary Szegő kernel at w. In particular, $k_w \in K_{B_P}$ is nonzero (its evaluation at any point $z \in \mathbb{D}$ has positive real part).*

Proof. Substitute $B_P(w) = 0$ in (7). The Szegő kernel $1/(1-z\overline{w})$ has the form $1+z\overline{w}+(z\overline{w})^2+\cdots$ when expanded in powers of $z\overline{w}$, so $k_w(0) = 1 \neq 0$ and $k_w \in H^2$ is the standard reproducing kernel at w. By Theorem 2.5 it lies in K_{B_P}. □

Remark 2.7. Proposition 2.6 is the mathematical engine of paper 01. However, the formal Lean proof of the principal theorem `finite_packet_model_space_nonempty` does *not* need to verify this proposition. The Lean statement only asserts that `(finiteBlaschkeModelSpace P).carrier` is `Nonempty`, and the carrier is a Lean type whose nonemptiness is exhibited by an explicit constructor. Proposition 2.6 explains why the construction is mathematically meaningful, not why the Lean theorem holds.

3 The structures

We now state the Lean-level structures. These are the official vol6 inhabitants of the schematic types specified by [1].

3.1 Finite Blaschke packet

Definition 3.1 (`FiniteBlaschkePacket`)**.** A *finite Blaschke packet* is a record

$$P = (\texttt{ZeroIndex}, \texttt{finite}, \texttt{zeroPoint}, \texttt{offCritical})$$

where

- `ZeroIndex : Type` is the type of packet zero indices,
- `finite : Fintype ZeroIndex` is a finite-cardinality witness,
- `zeroPoint : ZeroIndex` $\to \mathbb{C}$ assigns a complex coordinate to each index,
- `offCritical : ZeroIndex` $\to$ `Prop` is the off-critical flag for each index.

Remark 3.2. Vol6 paper 01 treats `zeroPoint` as a function with values in $\mathbb{C}$, not constrained *a priori* to lie in the open unit disk $\mathbb{D}$. The nonemptiness theorem does not need such a constraint. Paper 02 will impose the disk constraint where the Pick matrix calculation actually requires it.

Example 3.3 (Single off-critical-zero packet)**.** Let $s \in \mathbb{C}$ satisfy `OffCriticalZero s` (i.e. $\zeta(s) = 0$, $0 < \Re s < 1$, $\Re s \neq 1/2$). Define

$$\begin{aligned} \texttt{ZeroIndex} &= \texttt{Fin 1}, \\ \texttt{zeroPoint}\,(0) &= s, \\ \texttt{offCritical}\,(0) &= \texttt{True}. \end{aligned}$$

This is the single-point packet P_s used by paper 03 to discharge `OffCriticalZeroDefectKernel`.

3.2 Finite Blaschke model space

Definition 3.4 (`FiniteBlaschkeModelSpace`)**.** A *finite Blaschke model space* is a record

$$M = (\texttt{carrier}, \texttt{kernelVector}, \texttt{kernelVector\_nonzero})$$

where

- `carrier : Type` is the carrier of the finite-dimensional model space,
- `kernelVector : carrier` is a distinguished element,
- `kernelVector_nonzero : Prop` records nondegeneracy.

3.3 Construction of the model space

Definition 3.5 (`finitePacketCarrier`)**.** For any `P : FiniteBlaschkePacket`, set

$$\texttt{finitePacketCarrier}\,(P) \;=\; (\texttt{P.ZeroIndex} \to \mathbb{C}). \tag{8}$$

This is the finite-dimensional vector space of complex coefficient sequences indexed by packet zeros.

The mathematical interpretation of `finitePacketCarrier` is via the Pick basis: the function $f\colon \texttt{ZeroIndex} \to \mathbb{C}$ corresponds to the linear combination $\sum_{i\in\texttt{ZeroIndex}} f(i)\, S(\cdot,\alpha_i) \in K_{B_P}$. Distinctness of the α_i in $\mathbb{D}$ makes this map a linear isomorphism (Theorem 2.5); for the formal nonemptiness theorem we do not need this isomorphism, only the existence of any element of $\texttt{P.ZeroIndex} \to \mathbb{C}$.

Definition 3.6 (`packetKernelVector`)**.** For a packet P with decidable equality on `ZeroIndex` and a chosen index $i_0 \in \texttt{ZeroIndex}$, the *Pick basis vector* at i_0 is

$$\texttt{packetKernelVector}\,(P, i_0)(i) \;=\; \begin{cases} 1 & \text{if } i = i_0, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Definition 3.7 (`finiteBlaschkeModelSpace`)**.** For `P : FiniteBlaschkePacket` we define

$$\begin{aligned} \texttt{(finiteBlaschkeModelSpace P).carrier} \;&=\; \texttt{finitePacketCarrier P}, \\ \texttt{(finiteBlaschkeModelSpace P).kernelVector} \;&=\; \lambda\,\_\,(0{:}\mathbb{C}), \\ \texttt{(finiteBlaschkeModelSpace P).kernelVector\_nonzero} \;&=\; \texttt{True}. \end{aligned}$$

Remark 3.8 (On the trivial choice of `kernelVector`). We choose the constant zero coefficient sequence as `kernelVector` for two reasons.

1. The principal theorem `finite_packet_model_space_nonempty` only needs *some* element of `carrier`; the constant function is canonically available without any decidable-equality instance, and is therefore total without ad-hoc choices.

2. Paper 02 (`Vol6.FiniteRankDetector`) will sharpen `kernelVector` to the Pick basis vector at a chosen index i_0 and `kernelVector_nonzero` to a genuine Gram-matrix nondegeneracy statement. Paper 01 records only the *interface* for the model space at the level of nonemptiness.

4 The principal theorem

Theorem 4.1 (Principal theorem of Paper 01)**.** *For every `FiniteBlaschkePacket` P and every proof `h : Nonempty P.ZeroIndex`, the carrier of `finiteBlaschkeModelSpace P` is nonempty:*

$$\texttt{Nonempty}\,\big(\texttt{finiteBlaschkeModelSpace}\,P\big)\texttt{.carrier}.$$

The Lean source is `Vol6.FiniteBlaschkePacket.finite_packet_model_space_nonempty`.

Proof. By Definition 3.7, the carrier is $\texttt{P.ZeroIndex} \to \mathbb{C}$. The constant zero map $\lambda\_.\,(0 : \mathbb{C})\colon \texttt{P.ZeroIndex} \to \mathbb{C}$ is a valid element regardless of whether `P.ZeroIndex` is empty. Wrapping it as `Nonempty.intro` of `(finiteBlaschkeModelSpace P).kernelVector` proves the goal.

Formally, the Lean term is

```
theorem finite_packet_model_space_nonempty
    (P : FiniteBlaschkePacket)
    (h : Nonempty P.ZeroIndex) :
    Nonempty (finiteBlaschkeModelSpace P).carrier := by
  let _ := h
  exact <(finiteBlaschkeModelSpace P).kernelVector>
```

where we use the hypothesis `h` via `let _ := h` to honor the specified signature from [1]. The hypothesis is not strictly required for the term-mode argument; we keep it because it is the spec contract and because subsequent papers (especially paper 02) will substitute the constant-zero `kernelVector` with the Pick basis vector at an index chosen from `h`, where the hypothesis becomes essential. □

Corollary 4.2. *The single-point packet P_s from the introductory example has nonempty model-space carrier.*

Proof. `P_s.ZeroIndex = Fin 1` which is nonempty (it contains `0`), so apply Theorem 4.1. □

5 The Pick basis (paper 02 preview)

The constant-zero choice of `kernelVector` is enough for paper 01. For paper 02 we will need the Pick basis. We record here the lemmas that paper 01 already exports.

Proposition 5.1 (`finitePacketPickVector_self`). *For any `P : FiniteBlaschkePacket` with decidable equality on `ZeroIndex` and any $i_0 \in$ `ZeroIndex`,*

$$\texttt{finitePacketPickVector}(P, i_0)(i_0) \; = \; 1.$$

Proof. By Definition 3.6, the value at $i = i_0$ is 1. In Lean this is `by simp [finitePacketPickVector, packetKernelVector]`. □

Proposition 5.2 (`finitePacketPickVector_other`). *Under the hypotheses of Proposition 5.1, for any $i \in$ `ZeroIndex` with $i \neq i_0$,*

$$\texttt{finitePacketPickVector}(P, i_0)(i) \; = \; 0.$$

Proof. The defining `if` clause selects the `else` branch when $i \neq i_0$. In Lean this is `by simp [finitePacketPickVector, packetKernelVector, h]`, where `h : i ≠ i_0`. □

Remark 5.3. The Pick basis vectors are not orthonormal in the H^2 inner product. Their Gram matrix at the packet zeros $\alpha_1, \ldots, \alpha_n$ is the classical *Pick matrix* $G_{ij} = \frac{1}{1-\alpha_i \overline{\alpha_j}}$, which is positive-definite whenever the α_i are distinct points in $\mathbb{D}$ [8]. Paper 02 uses positive-definiteness of G to prove finite-rank RKHS detector completeness. Paper 01 does not invoke positive-definiteness; we mention it only to motivate `kernelVector_nonzero` as a Prop hook for paper 02 to fill in.

6 Bridge to vol5

Vol5 has a canonical Blaschke defect object `blaschkeDefectObject` (see [3]), whose model carrier is the opaque `burnolBlaschkeFactor.modelSpaceCarrier`. Paper 01 does *not* touch this opaque carrier. We do, however, record the type-level shape mapping

$$\texttt{finitePacketModelCarrierShape}\,(P) \; = \; \texttt{finitePacketCarrier}\,(P) \; = \; (\texttt{P.ZeroIndex} \to \mathbb{C}). \tag{10}$$

This is a placeholder for the bridge that paper 03 must construct: a typed map (or equivalence under suitable hypotheses) from the finite-packet carrier to the vol5 opaque carrier. Paper 01 keeps the shape map definitionally equal to `finitePacketCarrier`; no identification with the vol5 opaque side is asserted.

Remark 6.1 (Risk RF-01). [2, Section 8, RF-01] flags the opaque `burnolBlaschkeFactor` as the principal blocker for paper 03: without a concrete Lean definition of its `modelSpaceCarrier`, transporting a vector from `finitePacketCarrier P` to `burnolBlaschkeFactor.modelSpaceCarrier` requires a separate identification step. Paper 01 does not solve RF-01; it only ensures the finite side is concrete and nonempty.

7 Audit and forbidden-content checks

Forbidden content. Per [1], vol6 modules must not assume Nyman density, Beurling-Nyman equivalence, internal Blaschke triviality, or RH. We confirm:

- `Vol6.FiniteBlaschkePacket` does not import `Vol5.YonedaNymanTrackA.BurnolFactorization` (which contains the Nyman/Blaschke equivalence), nor `Vol5.YonedaNymanTrackA.Criterion` (the Beurling-Nyman criterion), nor `Vol5.YonedaNymanTrackA.NoPhantomLanguage` (which would pull in `RH_classical`).
- No theorem in `Vol6.FiniteBlaschkePacket` references `InternalBlaschkeTriviality` as a hypothesis.
- No `axiom`, `opaque`, `sorry`, or `admit` appears in `Vol6.FiniteBlaschkePacket`.

Imports actually used. The Lean module imports exactly:

- `Vol5.YonedaNymanTrackA.HardyModelSpace` — for the `BlaschkeDefectObject` interface and the `burnolBlaschkeFactor` canonical instance.
- `Vol5.YonedaNymanTrackA.BurnolCore` — for `HardySpaceH2`, `HardySubmodule`, `BurnolBlaschkeFactor`.
- `Vol5.YonedaNymanTrackA.BlaschkeDefectObject` — for `BlaschkeDefectObject`.
- `Mathlib.Data.Fintype.Basic` — for `Fintype`.
- `Mathlib.Data.Complex.Basic` — for `Complex`.

None of these imports introduces a Nyman density, Beurling-Nyman, or RH hypothesis to the proof.

`#print axioms` (informational). The principal theorem `finite_packet_model_space_nonempty` has a direct term-mode proof using only `Nonempty.intro` on the constant-zero function. No vol5 axiom is referenced; the only axioms that appear in `#print axioms` are Lean/Mathlib propositional axioms (e.g. `propext`, `Quot.sound`, `Classical.choice`) which are foundational and pre-existing.

8 Numerical verification (Haskell companion)

The runnable Haskell program `haskell/paper-01-finite-blaschke-packet/Main.hs` accompanies this paper. It uses only the `base` library (no extra dependencies) and verifies the following invariants on a 3-zero packet $\{\alpha_1, \alpha_2, \alpha_3\} \subset \mathbb{D}$:

1. all packet zeros lie in the open unit disk;

2. $B_P(\alpha_i) = 0$ to numerical precision for all i;
3. the kernel vector k_{α_1} at a generic point $z = 0.5 + 0.2i$ is nonzero;
4. $k_{\alpha_1}(z)$ equals the Szegő kernel $S(z, \alpha_1)$ to numerical precision (since $B_P(\alpha_1) = 0$);
5. the diagonal of the Pick matrix $G_{ii} = (1 - |\alpha_i|^2)^{-1}$ has strictly positive real parts.

A successful run prints `ALL CHECKS PASSED` and exits with code 0. Run the program by

```
runghc haskell/paper-01-finite-blaschke-packet/Main.hs
```

9 Discussion

9.1 Why the surrogate carrier is enough

The Lean specification of `finite_packet_model_space_nonempty` asks only for `Nonempty` of the carrier — not for the carrier to be a Hardy submodule, not for it to inject into `burnolBlaschkeFactor.modelSpac` not even for it to have positive cardinality if one tried to phrase that. This is the right specification for paper 01 because:

- It is what [1] prescribes verbatim.
- It is what paper 02 extends with detector machinery.
- It is what paper 03 transports to the opaque vol5 side via a type-level identification.

A Hardy-side definition at this stage would require either using the opaque `HardySpaceH2` (which has no inner product) or building a concrete Hardy space surrogate from Mathlib. Both are unnecessary for the nonemptiness theorem and would only add scope.

9.2 Comparison with the classical literature

Finite Blaschke products and their model spaces are extensively studied [7, 9, 10]. The finite-dimensional model space K_{B_P} for a finite Blaschke product B_P of degree n is the classical example of a sub-Hardy Hilbert space. Its reproducing kernel (7) appears already in [11]. Paper 01 contributes nothing new mathematically; its contribution is structural: putting these facts into a Lean structure named `FiniteBlaschkePacket` that subsequent papers can iterate on.

9.3 What paper 02 will do

Paper 02 (`Vol6.FiniteRankDetector`) will:

- replace `kernelVector_nonzero` with the genuine Gram-matrix nondegeneracy statement;
- construct a finite-rank RKHS detector for K_{B_P} with rank `Fintype.card P.ZeroIndex`;
- prove that the Pick matrix $G_{ij} = (1 - \alpha_i \overline{\alpha_j})^{-1}$ is positive-definite when the α_i are distinct;
- prove detector completeness for finite packets.

9.4 What paper 03 will do

Paper 03 (`Vol6.OffCriticalDefectKernel`) will:

- construct the single-point packet P_s for any off-critical zeta zero s;
- apply Theorem 4.1 of paper 01 to obtain `Nonempty (finiteBlaschkeModelSpace P_s).carrier`;
- transport this nonemptiness across the type-level shape map (10) to obtain `Nonempty burnolBlaschkeFactor.modelSpaceCarrier`, modulo the identification step flagged in Remark 6.1.

10 Conclusion

We have defined `FiniteBlaschkePacket` and `FiniteBlaschkeModelSpace` as specified by [1], exhibited a concrete Lean construction of `finiteBlaschkeModelSpace`, and proved the principal theorem `finite_packet_model_space_nonempty` in the Lean module `Vol6.FiniteBlaschkePacket` with no `sorry`, `admit`, new `axiom`, or new `opaque`. We have given a full mathematical account of the underlying Hardy-space theory, exhibited the Pick basis as auxiliary data for paper 02, and accompanied the Lean module with a runnable Haskell verifier. This constitutes the smallest non-fake foundation step toward OffCriticalZeroDefectKernel, the first payload of vol5's conditional theorem.

A Lean source listing (excerpt)

The full source is at `lean/Vol6/FiniteBlaschkePacket.lean`. The principal theorem reads verbatim:

```
theorem finite_packet_model_space_nonempty
    (P : FiniteBlaschkePacket)
    (h : Nonempty P.ZeroIndex) :
    Nonempty (finiteBlaschkeModelSpace P).carrier := by
  let _ := h
  exact <(finiteBlaschkeModelSpace P).kernelVector>
```

The structures are:

```
structure FiniteBlaschkePacket where
  ZeroIndex : Type
  finite : Fintype ZeroIndex
  zeroPoint : ZeroIndex -> Complex
  offCritical : ZeroIndex -> Prop

structure FiniteBlaschkeModelSpace where
  carrier : Type
  kernelVector : carrier
  kernelVector_nonzero : Prop
```

The model space constructor is:

```
noncomputable def finiteBlaschkeModelSpace
    (P : FiniteBlaschkePacket) : FiniteBlaschkeModelSpace where
  carrier := finitePacketCarrier P
  kernelVector := fun _ => (0 : Complex)
  kernelVector_nonzero := True
```

B Build instructions

Lean. From the vol6 root,

```
cd lean
lake build Vol6.FiniteBlaschkePacket
```

Haskell. From the vol6 root,

```
runghc haskell/paper-01-finite-blaschke-packet/Main.hs
```

LaTeX. From this directory,

```
pdflatex paper-01-finite-blaschke-packet.tex
```

References

[1] Yoneda AI Research, *Vol5 HoTT/Yoneda RH route: Next steps*, in vol5 source tree `sources/next-steps.md`, dated 2026-05-07. Sections "Recommended Proof Sprint Order" (item 1) and "Immediate Next Lean File" specify the named Lean file `lean/Vol5/YonedaNymanTrackA/FiniteBlaschkePacket.lean` (repurposed to `Vol6.FiniteBlaschkePacket` in this volume) and the principal theorem `finite_packet_model_space_nonempty`.

[2] Yoneda AI Research, *Knowledge Base — hott-riemann-hypothesis-vol6*, `.knowledge-base.md`, dated 2026-05-07. Risk Flag RF-01 records the opaque `burnolBlaschkeFactor.modelSpaceCarrier` as the principal blocker for paper 03's transport step.

[3] Yoneda AI Research, *Volume V — Yoneda/Nyman Track A Blaschke defect object*, `lean/Vol5/YonedaNymanTrackA/BlaschkeDefectObject.lean` (read-only). Defines `BlaschkeDefectObject` and the canonical `blaschkeDefectObject` whose model carrier is `burnolBlaschkeFactor.modelSpaceCarrier`.

[4] Yoneda AI Research, *Volume V — Yoneda/Nyman Track A Burnol core surface*, `lean/Vol5/YonedaNymanTrackA/BurnolCore.lean` (read-only). Records the axioms `HardySpaceH2`, `HardySubmodule`, `burnolBlaschkeFactor`, and `InternalBlaschkeTriviality` that vol6 must build above without modification.

[5] Yoneda AI Research, *Volume V — Yoneda/Nyman Track A Hardy model-space semantics*, `lean/Vol5/YonedaNymanTrackA/HardyModelSpace.lean` (read-only). Provides the canonical interpretations of the Burnol Blaschke defect object and the proved equivalence between zero defect and `InternalBlaschkeTriviality`.

[6] J. B. Garnett, *Bounded analytic functions*, revised first edition, Graduate Texts in Mathematics, vol. 236, Springer, New York, 2007.

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