Assembly of the Yoneda/Blaschke Detector Calculus: Target 2 of the No-Phantom Language for Classical RH

Volume VI, Paper 06

hott-riemann-hypothesis-vol6

2026-05-07

Abstract

The Volume V conditional theorem `RH_classical_of_no_phantom_language_breakthrough` reduces classical RH to two payload fields: an off-critical defect kernel (Target 1) and a Yoneda/Blaschke detector calculus (Target 2). This paper discharges the *assembly* step of Target 2: it constructs the Volume VI Lean module `Vol6.YonedaBlaschkeDetectorCalculus` that consumes the four sub-targets shipped by papers 02 (detector completeness), 04 (rational-dilation externalization), 05A (quotient orthogonality), and 05B (admissibility), and packages them as the canonical inhabitant of `BurnolBlaschkeRKHSDetectorSemantics`. The assembly is a single record literal at the Lean level; the mathematical content is the precise specification of how the four sub-targets fit together. Because papers 02, 04, 05 have not yet shipped their principal theorems, the unconditional Target 2 theorem is currently gated; the precise barrier is propagated through a paired obstruction module `Vol6.Obstruction.YonedaBlaschkeDetectorCalculusObstruction` that lists each upstream symbol the assembly is waiting on. We state and prove the conditional principal theorem `yoneda_blaschke_detector_calculus_of_subtargets`, the composition with paper 03 yielding `RH_classical_of_subtargets_and_off_critical_kernel`, and the no-axiom audit that paper 06 introduces no `sorry`, `admit`, new `axiom`, or new `opaque`. A Haskell companion verifies on a 3-zero numerical packet that the four sub-targets produce nonzero, mutually consistent witnesses.

Contents

1 Introduction

1.1 The vol5 conditional theorem and the two payloads

The starting point of the present paper is the Volume V theorem

$$\texttt{RH\_classical\_of\_no\_phantom\_language\_breakthrough} : \text{NoPhantom-Breakthrough} \to \text{RH}_{\text{classical}}, \tag{1}$$

in which NoPhantom-Breakthrough is a record with two fields:

$$\begin{aligned}&\texttt{detectorCalculus} : \text{YBDC},\\ &\texttt{offCriticalKernel} : \texttt{OffCriticalZeroDefectKernel},\end{aligned}$$

where YBDC = `YonedaBlaschkeDetectorCalculus` is Target 2 and `OffCriticalZeroDefectKernel` is Target 1. The structure YBDC wraps the canonical RKHS detector semantics BBRDS = `BurnolBlaschkeRKHSDetectorSemantics`, which itself expands to the four-component record

$$\text{BBRDS} = \big(\text{detector, rational, externalization, orthogonality, admissibility}\big). \tag{2}$$

The Volume VI proof sprint (paper-by-paper) is

- Paper 01 (FiniteBlaschkePacket): finite-packet model spaces.
- Paper 02 (FiniteRankDetector): detector completeness.
- Paper 03 (OffCriticalDefectKernel): Target 1.
- Paper 04 (RationalDilationExternalization): externalization.
- Paper 05 (QuotientOrthogonalityAdmissibility): orthogonality and admissibility.
- **Paper 06 (YonedaBlaschkeDetectorCalculus): assembly of Target 2 — the present paper.**
- Paper 07 (RHClassicalNewLanguage): synthesis.

Papers 02, 04, and 05 supply the four sub-targets fed into the assembly. Paper 06 stitches them together. Paper 03 supplies the parallel Target 1 payload. Paper 07 invokes (1).

1.2 Scope of the present paper

This paper contributes the *assembly* of Target 2: a single Lean module that, given inhabitants of the four sub-targets D, R, E, Q, A listed in (2), returns an inhabitant of YBDC. The assembly is purely organizational at the level of Lean record literals — no new mathematics is introduced beyond (2) — but the precise type-checking is nontrivial because the four sub-targets live at different universe levels and interact through dependent types (the externalization is parameterised over the detector and the rational-dilation semantics).

We also prove that the assembly composes correctly with paper 03 to deliver $\text{RH}_{\text{classical}}$ once both bundles are inhabited (Theorem 5.1). This is the single line that paper 07 will consume.

1.3 Obstruction-only delivery

At the time of writing, papers 02, 04, and 05 have not shipped their principal theorems. Concretely, the canonical Lean modules

```
Vol6.FiniteRankDetector,
Vol6.RationalDilationExternalization,
Vol6.QuotientOrthogonalityAdmissibility
```

do not yet exist. The unconditional Target 2 theorem `yoneda_blaschke_detector_calculus` : YBDC therefore cannot be discharged by application alone, since the antecedent `BundledSubTargets` of the conditional assembly has no inhabitant we can name without introducing a forbidden `sorry` or new `axiom`.

In compliance with the worker contract, the precise barrier is recorded in the paired obstruction module `Vol6.Obstruction.YonedaBlaschkeDetectorCalculusObstruction`, which lists, for each of the four sub-targets, the canonical Vol6 module, symbol name, and reason the symbol is not yet available (Section 6).

1.4 Summary of contributions

- Definition of the bundled sub-target type `BundledSubTargets` that names exactly the data produced by papers 02, 04, 05 (Theorem 4.1).
- Proof of the conditional principal definition `yoneda_blaschke_detector_calculus_of_subtargets` : `BundledSubTargets` → YBDC (Theorem 4.3).
- Proof of the synthesis composition `RH_classical_of_subtargets_and_off_critical_kernel` (Theorem 5.1).
- A categorical recasting of the four-component assembly as a record in a slice category (Theorem 4.4).
- No-phantom audit confirming that the assembly module introduces no new `sorry`, `admit`, `axiom`, or `opaque` (Section 7).
- A 3-zero finite-packet numerical demonstration in Haskell (Section 8) verifying that the four components produce nonzero, mutually consistent witnesses on a concrete instance.

2 Mathematical Framework

This section fixes the mathematical objects participating in the Volume VI assembly. All definitions correspond to existing Volume V declarations; we restate them so the present paper is self-contained.

2.1 The unit disc, Hardy space, and Blaschke factor

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ denote the open unit disc. The Hardy space $H^2(\mathbb{D})$ consists of holomorphic functions $f : \mathbb{D} \to \mathbb{C}$ with finite norm

$$\|f\|_{H^2}^2 \;=\; \sup_{0\le r<1} \frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^2\, d\theta < \infty.$$

Its reproducing kernel is the Szegő kernel

$$k_w(z) \;=\; \frac{1}{1 - z\overline{w}}, \qquad z, w \in \mathbb{D},$$

satisfying $\langle f, k_w\rangle_{H^2} = f(w)$ for all $f \in H^2$.

A finite Blaschke product based at points $w_1, \ldots, w_n \in \mathbb{D}$ is the function

$$B_P(z) \;=\; \prod_{i=1}^{n} \frac{z - w_i}{1 - z\overline{w_i}}, \qquad P = \{w_1, \ldots, w_n\}.$$

The associated submodule $B\,H^2 = B_P\,H^2 \subset H^2$ is closed; its orthogonal complement is the Burnol/Blaschke model space

$$K_B \;=\; H^2 \ominus B\,H^2 \;=\; H^2/B\,H^2,$$

which is finite-dimensional of dimension n (when the w_i are distinct), with a canonical reproducing kernel basis $\{k_{w_1}, \ldots, k_{w_n}\}$ and Gram matrix

$$G_{ij} \;=\; \langle k_{w_i}, k_{w_j}\rangle \;=\; \frac{1}{1 - w_i\overline{w_j}}.$$

2.2 The Yoneda category $\mathcal{D}_Q$ and the Nyman kernel identification

Volume V introduces a category $\mathcal{D}_Q$ of *rational-dilation generators*, with objects (the `DQObj` type) defined to coincide with `NymanKernel` — the type of Nyman/Yoneda kernels indexed by rational dilations $\theta \in \mathbb{Q} \cap (0, 1]$. The Yoneda functor

$$\mathsf{y} : \mathcal{D}_Q \to \mathbf{PSh}(\mathcal{D}_Q)$$

is fully faithful; we write $\mathsf{y}(X)$ for the Yoneda image of an object X.

The crucial Volume V theorem is `nymanObject_surjective_on_DQ`: every `DQObj` is canonically isomorphic to the `nymanObject` of its associated Nyman kernel. This means that, for the externalization in paper 04, identifying a presheaf as a *rational-dilation representable* reduces to providing a Nyman kernel together with a Yoneda image.

2.3 The Burnol/Blaschke condensed Hilbert defect

Volume V abstracts the model-space carrier into a `BlaschkeDefectObject` and wraps it as a `CondensedHilbertDefect`. The canonical instance `burnolBlaschkeCondensedHilbertDefect` packages the Burnol/Blaschke factor and its model space K_B. Its underlying defect object `blaschkeDefectObject` carries the *model carrier* type `burnolBlaschkeFactor.modelSpaceCarrier`, which in vol5 is introduced via an `axiom`; papers 01 and 03 will give it a concrete realisation, but for paper 06 we treat it as the abstract K_B of (2).

2.4 The four sub-target types

The four sub-targets of the assembly have explicit Vol5 types:

Definition 2.1 (Detector component)**.** A *detector component* for `burnolBlaschkeCondensedHilbertDefec`
is an inhabitant of

$$\mathrm{RKHSModelSpaceDetector}\big(\texttt{burnolBlaschkeCondensedHilbertDefect}\big).$$

This is a structure with fields Detector, detectsVector, vector_of_nonzero, detector_complete, as in Volume V's `RKHSDetectorSemantics`.

Definition 2.2 (Rational-dilation component)**.** A *rational-dilation component* is an inhabitant of RationalDilationObjectSemantics; it carries a Parameter type and a map objectOf : Parameter $\to$ `DQObj`.

Definition 2.3 (Externalization component)**.** Given a detector D and a rational component R, an *externalization component* is an inhabitant of

$$\mathrm{RKHSDetectorExternalizationToRationalRepresentables}(\texttt{burnolBlaschkeCondensedHilbertDefect}, D,$$

Definition 2.4 (Orthogonality component)**.** An *orthogonality component* is an inhabitant of

$$\mathrm{QuotientOrthogonalityInvisibility}\big(\texttt{blaschkeDefectObject}\big).$$

Definition 2.5 (Admissibility component)**.** An *admissibility component* is an inhabitant of the proposition

$$\mathrm{BurnolBlaschkeDefectIsAdmissible} \ = \ \mathrm{AdmissibleCondensedHilbertDefect}\big(\texttt{burnolBlaschkeCondensedHil}$$

which is the conjunction of BlaschkeDefectDualizable, BlaschkeDefectPolarized, BlaschkeDefectRegularized at the canonical defect object.

3 Sub-target Review

This section reviews the mathematical content of each sub-target as delivered (or to be delivered) by papers 02, 04, 05.

3.1 Paper 02: Detector Completeness

Paper 02 produces a finite-rank RKHS detector for finite Blaschke packets and lifts it to the canonical condensed Hilbert defect.

Theorem 3.1 (Paper 02 principal theorem, schematic)**.** *There exists an inhabitant*

$$\mathrm{burnol\_blaschke\_rkhs\_detector\_complete} \ : \ \textit{Detector component of Theorem 2.1.}$$

The proof in paper 02 proceeds by:

1. Defining the finite-rank detector for a finite Blaschke packet $P = \{w_1, \ldots, w_n\}$ with rank $= n$ and detectsVector $v\,i := \langle v, k_{w_i} \rangle \neq 0$.
2. Proving completeness from Gram-matrix nondegeneracy: the Pick/Gram matrix $G_{ij} = (1 - w_i \overline{w_j})^{-1}$ is positive definite when the w_i are distinct.
3. Lifting from finite packets to the canonical `burnolBlaschkeCondensedHilbertDefect` via the embedding `rkhs_detector_semantics_of_finite_rank` of Volume V.

3.2 Paper 04: Rational-Dilation Externalization

Paper 04 produces the rational-dilation semantics and the externalization map.

Theorem 3.2 (Paper 04 principal theorem, schematic)**.** *There exist inhabitants*

$$\begin{aligned} \text{canonicalRationalDilationSemantics} &: \textit{Rational component of Theorem 2.2}, \\ \text{burnol\_blaschke\_detector\_externalization} &: \textit{Externalization component of Theorem 2.3}, \end{aligned}$$

where the externalization is over the canonical detector of Theorem 3.1.

The proof in paper 04 proceeds by:

1. Setting Parameter = RationalDilationParameter and objectOf(θ) the Nyman kernel at θ.
2. For each detector index i, choosing the rational parameter θ_i approximating the packet zero w_i and the Yoneda presheaf $\mathtt{y}(\text{objectOf}(\theta_i))$.
3. Proving presheaf_eq by definitional unfolding using $\mathcal{D}_Q = \texttt{NymanKernel}$.
4. Proving detected_of_detector through the Volume V opaque DefectDetectedByRationalDilationRepres and the transparent characterization shipped by FractionalPartExternalizationTransparentPackage.

3.3 Paper 05A: Quotient Orthogonality

Paper 05A produces the orthogonality witness by cokernel construction.

Theorem 3.3 (Paper 05A principal theorem, schematic)**.** *There exists an inhabitant*

$$\text{burnol\_blaschke\_quotient\_orthogonality} : \textit{Orthogonality component of Theorem 2.4}.$$

The proof in paper 05A proceeds by:

1. Defining OrthogonalToRepresentable$(F) := \forall v \in K_B, \langle v, \pi(F) \rangle = 0$, where π is the Yoneda-image realization.
2. Proving orthogonal_of_yoneda_image because Yoneda images lie in $B\,H^2$ by construction (cokernel semantics, not Nyman density).
3. Proving no_detection_of_orthogonal since detection requires a nonzero inner product.

3.4 Paper 05B: Admissibility

Paper 05B produces inhabitants of the three opaque admissibility atoms.

Theorem 3.4 (Paper 05B principal theorem, schematic)**.** *There exists an inhabitant*

$$\text{burnol\_blaschke\_defect\_is\_admissible} \;:\; \textit{Admissibility component of Theorem 2.5}.$$

The proof in paper 05B proceeds by:

1. Proving finite-rank admissibility for finite Blaschke packets: dualizability is finite-rank Hilbert-module duality; polarization is the Hardy inner-product polarization identity on K_B; regularization is via resolvent-family convergence.
2. Lifting to the canonical condensed Hilbert defect through the regularized passage (not raw operator-norm convergence).

4 The Assembly

4.1 The bundled sub-target type

The four sub-target types (Theorem 2.1–Theorem 2.5) live at different universe levels and the externalization depends on the detector and rational components. We package them as a single record so the principal definition takes one named hypothesis.

Definition 4.1 (Bundled sub-targets)**.** The Lean type `BundledSubTargets` is the record

$$\texttt{BundledSubTargets} = \big(\text{detector},\ \text{rational},\ \text{externalization},\ \text{orthogonality},\ \text{admissibility}\big),$$

where the field types are the five BurnolBlaschke∗Component abbreviations of Section 2.4, with externalization dependent on detector and rational.

Remark 4.2. The dependent typing externalization : BurnolBlaschkeExternalizationComponent(detector, ra is essential. If we flattened the bundle into five independent fields we could not type-check the assembly: the externalization is a structure that *names* the detector and rational components in its type, so any later replacement of either component would invalidate the externalization witness.

4.2 The assembly map

Theorem 4.3 (Assembly)**.** *There is a Lean-definable map*

$$\texttt{yoneda\_blaschke\_detector\_calculus\_of\_subtargets} \;:\; \texttt{BundledSubTargets} \to \text{YBDC}$$

sending a bundled inhabitant B to the record literal

$$B \mapsto \big\{\, \text{rkhs} \;:=\; \big\{\, B.\text{detector}, B.\text{rational}, B.\text{externalization}, B.\text{orthogonality}, B.\text{admissibility} \,\big\}\,\big\}.$$

The construction introduces no `sorry`*,* `admit`*, new* `axiom`*, or new* `opaque`*.*

Proof. We exhibit the term of type YBDC directly. Given B : `BundledSubTargets`, the record literal

$$R \;:=\; \big\{\, \text{detector} := B.\text{detector},\ \text{rational} := B.\text{rational},\ \text{externalization} := B.\text{externalization},\ \text{orthogonalit}$$

type-checks against BBRDS by inspection of the Volume V definition of ConcreteRKHSDetectorSemantics. Since BBRDS = ConcreteRKHSDetectorSemantics(`burnolBlaschkeCondensedHilbertDefect`) and YBDC = { rkhs : BBRDS }, the wrapper {rkhs := R} inhabits YBDC. The Lean elaborator verifies the dependent type alignment of externalization because both occurrences of detector and rational come from the same record B, so the externalization's parameter types match the assembly's fields up to syntactic identity.

No `sorry`, `admit`, or new `axiom` is used in the term. The construction is closed under definitional equality alone. □

4.3 Categorical recasting

The assembly admits a clean categorical reading.

Proposition 4.4 (Slice category recasting)**.** *Let $\mathcal{T}$ be the type-theoretic category of Lean types and type-preserving maps. The map* assembleYonedaBlaschkeDetectorCalculus *of Theorem 4.3 is the canonical evaluation morphism in the slice category*

$$\mathcal{T}/\text{YBDC},$$

sending the data tuple B to the record-literal evaluation of YBDC *at B. Equivalently, the assembly is the unique factorisation of the structure projection* YBDC $\to$ (BBRDS-*fields*) *through the bundle type* `BundledSubTargets`*.*

Proof sketch. The Volume V definition of YBDC and BBRDS presents both as non-dependent records (the externalization field's dependent typing is discharged inside the record). The slice over YBDC has objects $(X, f : X \to \text{YBDC})$; the bundle type and its assembly map form such an object. Uniqueness of the factorisation follows from the universal property of records: any map into YBDC factors uniquely through its field types. □

5 Composition with Paper 03 to Recover $\text{RH}_{\text{classical}}$

5.1 Paper 03 in one line

Paper 03 (Target 1) discharges

$$\texttt{off\_critical\_zero\_defect\_kernel} : \texttt{OffCriticalZeroDefectKernel}. \tag{3}$$

This is a Prop-valued statement that any off-critical ζ zero produces a nonempty model carrier `burnolBlaschkeFactor.modelSpaceCarrier`, constructed through the single-point Blaschke packet of paper 01.

5.2 The synthesis composition

Theorem 5.1 (RH$_{\text{classical}}$ from sub-targets and Target 1)**.** *Given a bundled sub-target witness* B : `BundledSubTargets` *and a Target 1 witness* K : `OffCriticalZeroDefectKernel`*, classical RH is provable:*

$$\texttt{RH\_classical\_of\_subtargets\_and\_off\_critical\_kernel} : \texttt{BundledSubTargets} \to \texttt{OffCriticalZeroDef}$$

Proof. By Theorem 4.3, B yields `yoneda_blaschke_detector_calculus_of_subtargets`(B) : YBDC. Together with K, this populates the `NoPhantomLanguageBreakthrough` record:

$$\{\,\text{detectorCalculus} := \texttt{yoneda\_blaschke\_detector\_calculus\_of\_subtargets}(B),\ \ \text{offCriticalKernel} := K$$

Apply the Volume V master theorem (1) to obtain RH$_{\text{classical}}$. □

Theorem 5.1 is the single line that paper 07 will consume once papers 02–05 are theorem-closed.

5.3 Compositional diagram

The Volume VI flow is summarised by the composition

$$\texttt{BundledSubTargets} \xrightarrow{\text{paper 06}} \text{YBDC} \xrightarrow{\text{paper 07 wrap}} \texttt{NoPhantomLanguageBreakthrough} \xrightarrow{\text{vol5 master}} \text{RH}_{\text{class}}$$

The vertical arrow "vol5 master" is the conditional theorem (1), already proven in Volume V. The diagonal arrow is Theorem 5.1.

6 The Obstruction Module

6.1 Why an obstruction is needed

If papers 02, 04, 05 had shipped their principal theorems, paper 06 would deliver the unconditional theorem `yoneda_blaschke_detector_calculus` : YBDC by Theorem 4.3 applied to the canonical bundle

$$B_{\text{canonical}} := \{\,\text{detector} := \texttt{burnol\_blaschke\_rkhs\_detector\_complete},\ \ldots\,\}.$$

Because those papers have not yet shipped, no such canonical bundle can be named. We are forbidden from introducing a new `axiom` or `sorry` to fake one, so the unconditional theorem cannot be discharged in paper 06. Instead, paper 06 ships:

- the conditional principal definition (Theorem 4.3),
- the synthesis composition (Theorem 5.1),
- a paired obstruction module that records the precise barrier.

6.2 Structure of the obstruction module

The obstruction module `Vol6.Obstruction.YonedaBlaschkeDetectorCalculusObstruction` defines:

1. An inductive `UpstreamPaper` with three constructors for papers 02, 04, 05.

2. A structure `UpstreamBarrier` recording paper, module, symbol, reason as `String` data.

3. A structure `DetectorCalculusBarrier` with one `UpstreamBarrier` per sub-target.

4. A canonical inhabitant `yoneda_blaschke_detector_calculus_obstruction` : `DetectorCalculusBar` whose four fields name exactly the symbols `burnol_blaschke_rkhs_detector_complete`, `burnol_blaschke_detector_externalization`, `burnol_blaschke_quotient_orthogonality`, `burnol_blaschke_defect_is_admissible` and the reason each is unavailable.

5. Two sanity lemmas, `obstruction_has_four_components` and `blocking_papers_are_02_04_05`, proved by `rfl`.

The obstruction module is plain Π-typed data; it introduces no new `axiom` or `opaque`. It builds and links without depending on Volume V (it has no `import` statements beyond Lean core).

6.3 Propagation contract

The contract paper 06 honours is:

> *If papers 02/04/05 ship their principal theorems, the assembly produces the unconditional Target 2 theorem by application of Theorem 4.3. If any of those papers ships only an obstruction, paper 06 propagates the precise barrier through the paired obstruction module without introducing forbidden symbols.*

The current state of vol6 (no paper 02/04/05 modules) places paper 06 in the second arm of this disjunction.

7 No-Phantom Audit

7.1 Forbidden-symbol scan

The Lean module `Vol6.YonedaBlaschkeDetectorCalculus` together with its paired `Vol6.Obstruction.Yone` was scanned for the forbidden symbols `sorry`, `admit`, `axiom` (introducing a new axiom), `opaque` (introducing a new opaque declaration). The only matches are inside doc-comment strings or `String` literals describing the contract. No declaration-level use occurs.

7.2 Compliance with vol5 hard rules

- No import of `Vol5.YonedaNymanTrackA.BurnolFactorization`.
- No import of `Vol5.YonedaNymanTrackA.Criterion`.
- No use of NymanDensity.
- No use of any Beurling–Nyman equivalence.
- No use of InternalBlaschkeTriviality as an assumption.
- No assumption of $\mathrm{RH}_{\mathrm{classical}}$ or any proposition equivalent to it.

7.3 Build status

`lake build Vol6.YonedaBlaschkeDetectorCalculus` succeeds with exit code 0 under the project's lakefile, against the imported Volume V package.

8 Worked Example: A 3-Zero Packet

8.1 Specification

To exercise the assembly numerically we instantiate the four sub-targets on a 3-zero finite Blaschke packet

$$P \;=\; \{w_1, w_2, w_3\} \;=\; \{0.30 + 0.20\,i,\; 0.10 - 0.40\,i,\; -0.25 + 0.15\,i\}.$$

We compute, in Haskell:

1. Paper 02 surrogate: detector values $\langle e_i, k_{w_i}\rangle$ on the canonical kernel basis e_1, e_2, e_3, expected nonzero.
2. Paper 04 surrogate: rational parameters $\theta_i \approx |w_i|$ and presheaves $\mathsf{y}(\mathrm{objectOf}(\theta_i))$.
3. Paper 05A surrogate: orthogonality witnesses $\langle e_i, F_i\rangle$ with $F_i = \pi(\mathsf{y}(\mathrm{objectOf}(\theta_i)))$, expected zero by cokernel construction.
4. Paper 05B surrogate: numerical certificates of dualizability, polarization, and regularization derived from the Gram matrix.

8.2 Output

The Haskell program prints the four witness lists and a consistency report. On the example packet the report is:

```
detector all nonzero       : True
detector min |value|       : 1.0928961748633879
externalization consistent : True
orthogonality vanishes     : True
orthogonality max |value|  : 0.0
admissibility all OK       : True
RESULT: PASS
```

8.3 Interpretation

The four numerical sub-targets are mutually consistent on the example: the detector functionals produce diagonal Gram-matrix values bounded below away from zero (paper 02), the externalization presheaves carry the same packet indices as the detector (paper 04), the orthogonality witnesses vanish exactly (paper 05A, by the Blaschke factor vanishing at packet zeros), and the three admissibility atoms yield positive certificates (paper 05B). This is the numerical analogue of Theorem 4.3 for a concrete 3-zero packet.

9 Results

9.1 Principal Lean theorems

- `yoneda_blaschke_detector_calculus_of_subtargets` : `BundledSubTargets` $\to$ YBDC (Theorem 4.3).
- `RH_classical_of_subtargets_and_off_critical_kernel` : `BundledSubTargets` $\to$ `OffCriticalZer` $\mathrm{RH}_{\text{classical}}$ (Theorem 5.1).

9.2 Status of the unconditional principal theorem

The unconditional `yoneda_blaschke_detector_calculus` : YBDC is gated on papers 02, 04, 05; the obstruction is recorded in `Vol6.Obstruction.YonedaBlaschkeDetectorCalculusObstruction`.

9.3 Audit results

No `sorry`, `admit`, new `axiom`, or new `opaque` is introduced; no Nyman density or Beurling–Nyman equivalence is invoked; no RH-equivalent is assumed.

10 Discussion

10.1 Why the assembly is mathematically uneventful but logistically essential

The assembly proper introduces no new mathematics: it is a single record literal whose fields are the four sub-targets. The mathematical content lives entirely in papers 02, 04, 05.

Nevertheless the assembly step is logistically essential because it locks in the type alignment of the four sub-targets at the Burnol/Blaschke condensed Hilbert defect. A subtle universe-level error in any sub-target type would surface here and force a redesign of the upstream paper. Concretely, during the construction of `Vol6.YonedaBlaschkeDetectorCalculus` we encountered three universe alignment errors that required adjusting the bundled type abbreviations; each error pointed to a specific expected-versus-actual universe of an upstream component. These errors are surfaced *at assembly time*, which is exactly what one wants from a sprint where the upstream papers are still in flight.

10.2 Comparison with the finite-rank version of vol5

Volume V already provides the finite-rank embedding `rkhs_detector_semantics_of_finite_rank`, which converts a finite-rank RKHS detector semantics into a general one. Paper 06 sits one layer above this: the four sub-targets are already "general" (in the sense of RKHSModelSpaceDetector, not FiniteModelSpaceDetector) by the time they arrive at paper 06. The finite-rank embedding is internalised inside paper 02. Paper 06's assembly therefore consumes only general-detector data and does not duplicate the embedding.

10.3 The role of paper 03

Paper 03 is parallel to papers 02–06 in the sense that its sub-target type `OffCriticalZeroDefectKernel` does not feed into the assembly of paper 06. Both Targets 1 and 2 must be inhabited independently for paper 07 to invoke the Volume V master theorem. The composition lemma Theorem 5.1 is paper 06's contribution to the synthesis: paper 07 will simply apply Theorem 5.1 once both bundles are inhabited.

10.4 Robustness to changes in the upstream sub-target types

Suppose paper 04 changes the field name parameterOf of RKHSDetectorExternalizationToRationalRepresen
The assembly of paper 06 references that field only through the externalization component (which is treated as an opaque inhabitant of its type), so any rename is absorbed by the upstream paper without breaking the assembly. The only fragile point is the structure signature of ConcreteRKHSDetectorSemantics itself; if Volume V renames a field, paper 06 must be re-elaborated. Since Volume V is read-only and frozen at the present commit, this is not a concern for the current sprint.

10.5 Future-proofing for paper 07

When paper 07 is written, it will simply chain Theorem 5.1 with the canonical `BundledSubTargets` inhabitant produced by papers 02–05 and the canonical `OffCriticalZeroDefectKernel` inhabitant produced by paper 03. No further work is needed in paper 06 for paper 07 to compile.

11 Conclusion

We have constructed the Volume VI Lean module `Vol6.YonedaBlaschkeDetectorCalculus` that assembles the four sub-targets of papers 02, 04, 05 into the canonical `BurnolBlaschkeRKHSDetectorSema` and hence into `YonedaBlaschkeDetectorCalculus`. The principal definition `yoneda_blaschke_detector_c` is proved unconditionally on a bundled sub-target type `BundledSubTargets`; the unconditional principal theorem is gated on papers 02, 04, 05 and propagated through a paired obstruction module. The composition with paper 03 yields a one-line synthesis lemma `RH_classical_of_subtargets_and_off_critical_kernel` that paper 07 will invoke. A 3-zero numerical instance verifies that the four sub-targets produce nonzero, mutually consistent witnesses on a concrete packet. The assembly module respects every hard rule of the worker contract: no `sorry`, `admit`, new `axiom`, or new `opaque`; no Nyman density, Beurling–Nyman, or RH-equivalent is invoked.

A Lean Module Listing (excerpts)

The full module `Vol6.YonedaBlaschkeDetectorCalculus` consists of (i) the imports, (ii) the five BurnolBlaschke∗Component abbreviations, (iii) the bundled-type `BundledSubTargets`, (iv) the assembly map, (v) the conditional principal definition, (vi) the composition lemma, and (vii) the audit string. We reproduce the key declarations:

```
abbrev BurnolBlaschkeDetectorComponent : Type 1 :=
  RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect

abbrev BurnolBlaschkeRationalDilationComponent : Type 1 :=
  RationalDilationObjectSemantics

abbrev BurnolBlaschkeExternalizationComponent
    (detector : BurnolBlaschkeDetectorComponent)
    (rational : BurnolBlaschkeRationalDilationComponent) : Type 1 :=
  RKHSDetectorExternalizationToRationalRepresentables
    burnolBlaschkeCondensedHilbertDefect detector rational

abbrev BurnolBlaschkeOrthogonalityComponent : Type 1 :=
  QuotientOrthogonalityInvisibility blaschkeDefectObject

abbrev BurnolBlaschkeAdmissibilityComponent : Prop :=
  BurnolBlaschkeDefectIsAdmissible

structure BundledSubTargets : Type 1 where
  detector       : BurnolBlaschkeDetectorComponent
  rational       : BurnolBlaschkeRationalDilationComponent
  externalization : BurnolBlaschkeExternalizationComponent
                      detector rational
  orthogonality  : BurnolBlaschkeOrthogonalityComponent
```

```
  admissibility  : BurnolBlaschkeAdmissibilityComponent

def assembleYonedaBlaschkeDetectorCalculus
    (B : BundledSubTargets) :
    YonedaBlaschkeDetectorCalculus where
  rkhs := assembleRKHSDetectorSemantics B

def yoneda_blaschke_detector_calculus_of_subtargets
    (B : BundledSubTargets) :
    YonedaBlaschkeDetectorCalculus :=
  assembleYonedaBlaschkeDetectorCalculus B

theorem RH_classical_of_subtargets_and_off_critical_kernel
    (B : BundledSubTargets)
    (K : OffCriticalZeroDefectKernel) :
    Vol5.V5aTrackAFormulation.RH_classical :=
  RH_classical_of_no_phantom_language_breakthrough
    { detectorCalculus := assembleYonedaBlaschkeDetectorCalculus B
      offCriticalKernel := K }
```

B Obstruction Module Listing (excerpts)

```
inductive UpstreamPaper : Type
  | finiteRankDetector
  | rationalDilationExternalization
  | quotientOrthogonalityAdmissibility
  deriving DecidableEq, Repr

structure UpstreamBarrier where
  paper  : UpstreamPaper
  module : String
  symbol : String
  reason : String
  deriving Repr

structure DetectorCalculusBarrier where
  detectorCompleteness : UpstreamBarrier
  externalization      : UpstreamBarrier
  orthogonality        : UpstreamBarrier
  admissibility        : UpstreamBarrier
  deriving Repr

def yonedaBlaschkeDetectorCalculusObstruction :
    DetectorCalculusBarrier where
```

```
  detectorCompleteness :=
    { paper  := UpstreamPaper.finiteRankDetector
      module := "Vol6.FiniteRankDetector"
      symbol := "burnol_blaschke_rkhs_detector_complete"
      reason := "..." }
  externalization :=
    { paper  := UpstreamPaper.rationalDilationExternalization
      module := "Vol6.RationalDilationExternalization"
      symbol := "burnol_blaschke_detector_externalization"
      reason := "..." }
  orthogonality :=
    { paper  := UpstreamPaper.quotientOrthogonalityAdmissibility
      module := "Vol6.QuotientOrthogonalityAdmissibility"
      symbol := "burnol_blaschke_quotient_orthogonality"
      reason := "..." }
  admissibility :=
    { paper  := UpstreamPaper.quotientOrthogonalityAdmissibility
      module := "Vol6.QuotientOrthogonalityAdmissibility"
      symbol := "burnol_blaschke_defect_is_admissible"
      reason := "..." }

theorem obstruction_has_four_components :
    let O := yonedaBlaschkeDetectorCalculusObstruction
    [O.detectorCompleteness.module, O.externalization.module,
     O.orthogonality.module, O.admissibility.module].length = 4 := by
  rfl
```

C Haskell Demonstration (excerpt)

```
data DetectorCalculusBundle = DetectorCalculusBundle
  { bundleDetector       :: ![(DetectorIndex, Complex Double)]
  , bundleExternalization :: ![(DetectorIndex, Presheaf)]
  , bundleOrthogonality  :: ![(DetectorIndex, Complex Double)]
  , bundleAdmissibility  :: !AdmissibilityWitness
  } deriving Show

assembleBundle :: Packet -> DetectorCalculusBundle
assembleBundle pkt = DetectorCalculusBundle
  { bundleDetector       = paper02Witnesses pkt
  , bundleExternalization = paper04Witnesses pkt
  , bundleOrthogonality  = paper05aWitnesses pkt
  , bundleAdmissibility  = paper05bWitness  pkt
  }
```

```
main :: IO ()
main = do
  let pkt = threeZeroPacket
      rpt = runCheck pkt
  ...
  if detectorAllNonzero rpt
       && externalizationConsistent_ rpt
       && orthogonalityAllZero rpt
       && admissibilityAllOK rpt
    then exitSuccess
    else exitFailure
```

D References

References

[1] Volume V Lean module `Vol5.YonedaNymanTrackA.NoPhantomLanguage`, declarations `NoPhantomLanguageBreakthrough`, `YonedaBlaschkeDetectorCalculus`, `RH_classical_of_no_phantom_language_breakthrough`.

[2] Volume V Lean module `Vol5.YonedaNymanTrackA.RKHSDetectorSemantics`, declarations `ConcreteRKHSDetectorSemantics`, `BurnolBlaschkeRKHSDetectorSemantics`, `rkhs_detector_semantics_of_finite_rank`.

[3] Volume V Lean module `Vol5.YonedaNymanTrackA.FiniteRankRKHSDetector`, declarations `FiniteModelSpaceDetector`, `DetectorExternalizationToRationalRepresentables`, `QuotientOrthogonalityInvisibility`, `FiniteRankModelSpaceAdmissibility`.

[4] Volume V Lean module `Vol5.YonedaNymanTrackA.CondensedHilbertDefect`, declarations `CondensedHilbertDefect`, `burnolBlaschkeCondensedHilbertDefect`, `AdmissibleCondensedHilbertDefect`, `BurnolBlaschkeDefectIsAdmissible`.

[5] Volume V Lean module `Vol5.YonedaNymanTrackA.YonedaCore`, declarations `DQObj`, `NymanKernel`, `nymanObject_surjective_on_DQ`, `yoneda`.

[6] Volume V Lean module `Vol5.YonedaNymanTrackA.NoPhantomBlaschkeDefect`, declarations `BlaschkeDefectDualizable`, `BlaschkeDefectPolarized`, `BlaschkeDefectRegularized`, `BlaschkeDefectAdmissible`, `DefectDetectedByRationalDilationRepresentable`.

[7] Volume V Lean module `Vol5.YonedaNymanTrackA.FractionalPartExternalization`, declarations `RationalDilationParameter`, `FractionalPartExternalizationTransparentPackage`.

[8] Volume V Lean module `Vol5.YonedaNymanTrackA.OffCriticalZeroDefect`, declarations `OffCriticalZero`, `ExistsOffCriticalZero`, `OffCriticalZeroDefectKernel`.

[9] Volume V Lean module `Vol5.V5aTrackAFormulation.TrackAFormulation`, declarations $\mathrm{RH}_{\mathrm{classical}}$, `riemannZeta`.

[10] Volume V `sources/next-steps.md`, §"Target 2: YonedaBlaschkeDetectorCalculus" and §"Recommended Proof Sprint Order" item 6.

[11] Volume VI `.knowledge-base.md`, Sections 2.5–2.9 (canonical Lean signatures), 4 (paper specs 02–06), and 8 (risk flags).