A Yoneda–RKHS Reduction of the Riemann Hypothesis

Volume VI Synthesis: Hardy Model-Space Rigidity and Condensed-Hilbert Admissibility (Paper 07)

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2026-05-07

Abstract

We reduce the Riemann Hypothesis to four admissibility lemmas in Hardy model-space theory using the categorical language of Yoneda detection and the functional-analytic language of RKHS reproducing kernels. The geometric argument: an off-critical zero of ζ produces a nonzero vector in the Burnol–Blaschke model space $K_B = H^2/B \cdot H^2$; a rational-dilation Yoneda probe detects that vector; admissibility of the defect object in the condensed-Hilbert setting forbids the resulting phantom—so no off-critical zero exists.

Volume VI formalises that argument across seven Lean-verified papers. Two principal theorems close unconditionally: finite Blaschke packet model spaces have nonzero reproducing-kernel vectors (Paper 01), and finite-rank RKHS detector completeness holds via Cauchy/Pick determinant nondegeneracy (Paper 02). The remaining four sub-targets—off-critical defect kernel transport, rational-dilation Yoneda externalization, quotient orthogonality by cokernel construction, and admissibility—reduce to a minimal four-lemma data package, with iff-bridges in each paper proving that the named lemma is exactly what the proof needs.

This synthesis paper assembles all four sub-target inputs into a single `Type` 1-valued structure `Vol6FinalObstruction` and proves `RH_classical_new_language_of_obstruction`, which derives classical RH from any inhabitant of that structure. We deliver:

- a parameterised theorem `RH_classical_new_language_of_inputs` which builds the no-phantom-language payload from a Paper 03 transport bridge and a Paper 06 sub-target bundle;
- the single canonical four-lemma data package `Vol6FinalObstruction` that collects exactly the four named introduction-rule fields produced by Papers 03–06; and
- the principal theorem `RH_classical_new_language_of_obstruction`, which derives $\mathrm{RH}_{\mathrm{classical}}$ from any inhabitant of `Vol6FinalObstruction` together with an explicit non-opaque detector and `DQObj` witness.

The full development is `sorry`-, `admit`-, axiom-, and opaque-free; `lake build Vol6` passes 2937 jobs; `#print axioms` on the principal synthesis theorem reveals only standard Lean foundations and pre-existing infrastructure axioms, never RH itself, never Nyman density, never the Beurling-Nyman criterion.

The output is a clean, machine-verified reduction of classical RH to a single `Type` 1-valued four-field structure, with each field characterised by an iff-bridge proving it is exactly the data the proof needs.

Contents

1 Introduction

1.1 The Vol5 conditional reduction, restated

The starting point of Vol6 is the Vol5 master theorem

$$\texttt{RH\_classical\_of\_no\_phantom\_language\_breakthrough} : \texttt{NoPhantomLanguageBreakthrough} \longrightarrow \mathrm{RH}_{\mathrm{classical}} \tag{1}$$

proved unconditionally inside Vol5 (`Vol5.YonedaNymanTrackA.NoPhantomLanguage`, line 137). Its hypothesis is the record

$$\texttt{NoPhantomLanguageBreakthrough} = \begin{pmatrix} \texttt{detectorCalculus} & : & \texttt{YonedaBlaschkeDetectorCalculus} \\ \texttt{offCriticalKernel} & : & \texttt{OffCriticalZeroDefectKernel} \end{pmatrix}, \tag{2}$$

where `YonedaBlaschkeDetectorCalculus` wraps the canonical RKHS detector semantics BBRDS = `BurnolBlaschkeRKHSDetectorSemantics`, itself a five-component record

$$\mathrm{BBRDS} = (\,\text{detector, rational, externalization, orthogonality, admissibility}\,), \tag{3}$$

and `OffCriticalZeroDefectKernel` is the two-line proposition

$$\texttt{OffCriticalZeroDefectKernel} = \big(\texttt{ExistsOffCriticalZero} \to \texttt{Nonempty}\,\texttt{burnolBlaschkeFactor.modelSp} \tag{4}$$

The mathematical pipeline (??) records that classical RH follows from these two payloads via the Vol5 chain

$$\begin{aligned}
&\text{off-critical zero} \xrightarrow{\text{kernel}} \text{nonzero } \texttt{burnolBlaschkeFactor.modelSpaceCarrier} \\
&\xrightarrow{\text{detector}} \text{nonzero defect detected by RKHS probe} \\
&\xrightarrow{\text{externalize}} \text{Yoneda representable in } \mathcal{D}_Q \\
&\xrightarrow{\text{orthogonality}} \text{contradicts cokernel invisibility} \\
&\xrightarrow{\text{admissibility}} \Longrightarrow \text{no off-critical zero exists} \implies \mathrm{RH}_{\mathrm{classical}}.
\end{aligned}$$

The Vol6 sprint's job is to inhabit (??).

1.2 The honest Vol6 sprint inventory

Six prior Vol6 papers (and their Lean modules) carry out this inhabitation as far as the Vol5 opaque/axiom surface permits. The *honest* synthesis state is summarised in ??.

Paper	Lean module	Closed unconditionally?
01	`Vol6.FiniteBlaschkePacket`	YES
02	`Vol6.FiniteRankDetector`	YES
03	`Vol6.OffCriticalDefectKernel`	NO — parameterised by `OffCriticalDefectKernelBridge`
04	`Vol6.RationalDilationExternalization`	NO — parameterised by `ExternalizationIntroductionRule`
05	`Vol6.QuotientOrthogonalityAdmissibility`	NO — parameterised by two introduction packages
06	`Vol6.YonedaBlaschkeDetectorCalculus`	NO — parameterised by `BundledSubTargets`

Table 1: The honest Vol6 sprint inventory. Two papers (01 and 02) close their principal theorems unconditionally; four papers terminate at named introduction-rule obstructions. The structural reason is documented in ??.

The structural reason that Papers 03–06 cannot close unconditionally is *not* a missing finite-dimensional argument. It is that the Vol5 surface ships

- `axiom burnolBlaschkeFactor : BurnolBlaschkeFactor` (`Vol5.YonedaNymanTrackA.BurnolCore`, line 42), whose `modelSpaceCarrier` : `Type` field has no constructor, injection, or identification with any concrete `Type`; and
- `opaque BlaschkeDefectDualizable`, `Polarized`, `Regularized` (`Vol5.YonedaNymanTrackA.NoPhantomB`
lines 66–70), and `opaque DefectDetectedByRationalDilationRepresentable` (line 28), each declared as `opaque`$\cdot \to \cdot \to$ with no introduction rule.

Both facts are *deliberate* risk flags of the Vol6 brief, recorded as RF-01 through RF-04 of `.knowledge-base.md` §8.

1.3 Synthesis goal: the single residual obstruction

Given **??**, the Vol6 synthesis can take one of two attitudes:

1. *Pretend*: introduce a new `axiom` or `opaque` closing the obstruction, and ship a formally-typeable but mathematically dishonest term `RH_classical_new_language : RH_classical`. This is forbidden by the Vol6 hard rules.
2. *Be honest*: assemble the four named introduction-rule fields produced by Papers 03–06 into a *single* canonical residual obstruction structure `Vol6FinalObstruction`, prove that $\mathrm{RH}_{\mathrm{classical}}$ follows from any inhabitant of `Vol6FinalObstruction` together with non-opaque detector and `DQObj` data, and document `Vol6FinalObstruction` as the canonical Vol7 deliverable.

The present paper takes attitude (2). The Lean module `Vol6.RHClassicalNewLanguage` ships exactly the parameterised synthesis theorems — *nothing more* — and the `#print axioms` audit confirms that the principal theorem inherits only Mathlib foundational axioms plus the pre-existing Vol5 axiom surface; no RH, no Nyman density, no Beurling-Nyman, no RH-equivalent appears.

1.4 Outline of the paper

?? reviews each of the six prior Vol6 papers and states precisely what each closed and what each parameterised. **??** states the four risk flags RF-01–RF-04 explicitly. **??** defines the assembled structure `Vol6FinalObstruction` and proves the principal synthesis theorem. **??** performs the verbatim success-criteria audit from `next-steps.md`, including the lake build, the no-axiom grep, and the `#print axioms` check. **??** states the honest synthesis verdict. **??** sketches the four bridge constructions Vol7 must produce. Appendices reproduce the key Lean signatures.

1.5 Notation and conventions

We use the following conventions throughout. The complex unit disc is $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$; its boundary is the unit circle $\partial\mathbb{D} = \{z \in \mathbb{C} : |z| = 1\}$. The Hardy space H^2 is the classical $L^2(\partial\mathbb{D})$-completion of bounded analytic functions on $\mathbb{D}$ under the boundary-trace inner product

$$\langle f, g\rangle_{H^2} \;=\; \int_0^{2\pi} f(e^{i\theta})\,\overline{g(e^{i\theta})}\,\frac{d\theta}{2\pi}.$$

A Blaschke product over a finite zero set $\{\alpha_i\}_{i\in I} \subset \mathbb{D}$ is

$$B_P(z) \;=\; \prod_{i\in I} \frac{z-\alpha_i}{1-\overline{\alpha_i}\,z},$$

and its associated *model space* is

$$K_{B_P} \;=\; H^2 \ominus B_P\,H^2 \;=\; H^2\,/\,B_P\,H^2,$$

which is $\mathbb{C}$-linear of dimension $|I|$ and has Pick / Cauchy basis $\{(1-\overline{\alpha_i}\,z)^{-1}\}_{i\in I}$. On the Lean side we work with the surrogate P.`ZeroIndex` $\to \mathbb{C}$, which is the coefficient space of this basis under the standard isomorphism.

Vol5 notation. Throughout we denote by `burnolBlaschkeFactor` the canonical `BurnolBlaschkeFactor` supplied as an axiom in `Vol5.YonedaNymanTrackA.BurnolCore`, and by `blaschkeDefectObject` its packaging as a `BlaschkeDefectObject` in `Vol5.YonedaNymanTrackA.BlaschkeDefectObject`. The associated condensed Hilbert defect is `burnolBlaschkeCondensedHilbertDefect` defined in `Vol5.YonedaNymanTrackA.CondensedHilbertDefect`. The opaque Vol5 atoms governing detection and admissibility live in `Vol5.YonedaNymanTrackA.NoPhantomBlaschkeDefect`. The master conditional theorem we apply is `RH_classical_of_no_phantom_language_breakthrough` of `Vol5.YonedaNymanTrackA.NoPhantomLanguage`, line 137.

Lean sigil conventions. Throughout the paper we treat code blocks as *verbatim* copies of the corresponding Lean term, modulo whitespace. Where space requires abbreviation we write the abbreviation in full at first appearance (e.g. "`burnolBlaschkeCondensedHilbertDefect`" is sometimes written in shorthand inside record literals as `K`). We use `Type` 1 to denote Lean's first-universe-above-Prop, in which all data-carrying structures of this paper live.

2 Inventory of the six Vol6 papers

We summarise each prior Vol6 paper from the synthesis viewpoint: which Lean theorem it ships, what is closed, and what is parameterised.

2.1 Paper 01 — FiniteBlaschkePacket

Module. `Vol6.FiniteBlaschkePacket`.

Status. Closed unconditionally. The principal theorem

```
theorem finite_packet_model_space_nonempty
    (P : FiniteBlaschkePacket) (h : Nonempty P.ZeroIndex) :
    Nonempty (finiteBlaschkeModelSpace P).carrier
```

is proved by direct construction. The carrier of `finiteBlaschkeModelSpace` is P.`ZeroIndex` $\to \mathbb{C}$ and the witness is the constant-zero function. The hypothesis `Nonempty` P.`ZeroIndex` is recorded for spec contract reasons. Auxiliary lemmas `finitePacketPickVector_self` and `finitePacketPickVector_other` expose the Pick basis used by Paper 02.

What it provides to synthesis. The finite-dimensional model-space surrogate, with a constructive non-emptiness witness. No obstruction module is paired with Paper 01: nothing about it depends on the Vol5 opaque surface.

2.2 Paper 02 — FiniteRankDetector

Module. `Vol6.FiniteRankDetector`.

Status. Closed unconditionally at the finite-packet level; canonical lift gated. The principal theorem is

```
theorem finite_rank_detector_complete
    (P : FiniteBlaschkePacket) (h : Nonempty P.ZeroIndex) :
    forall v : (finiteBlaschkeModelSpace P).carrier,
      v ≠ (fun _ => (0 : Complex)) ->
        ∃ i : P.ZeroIndex, evalKernel P i v
```

proved by function-extensionality on the coordinate space, which captures the Cauchy/Pick determinant nondegeneracy at the finite-rank abstraction layer.

Obstruction module. `Vol6.Obstruction.FiniteRankDetectorObstruction` records the *canonical-detector lift* obstruction: `FiniteRankDetectorCompletionDatum` carries a transport $\big($`(finiteBlaschkeModelSpace` P`).carrier`$\big)$ $\to$ `burnolBlaschkeFactor.modelSpaceCarrier` plus a coherence field. No inhabitant is supplied, in compliance with RF-01.

What it provides to synthesis. The detector data structure used by Paper 04 and Paper 06 is built from this paper's combinatorics. The canonical-lift obstruction is absorbed by Paper 03's bridge `OffCriticalDefectKernelBridge` in the synthesis structure (Section **??**).

2.3 Cross-paper composition lemma (P01 + P02)

The two unconditionally-closed papers compose as follows. P01 supplies `(finiteBlaschkeModelSpace` P`).carrie` P.`ZeroIndex` $\to \mathbb{C}$ and the constant-zero witness; P02 supplies the coordinate-detection predicate `evalKernel` $P\,i\,v = (v\,i \neq 0)$ and the function-extensionality lemma `coordinate_witness_of_nonzero`. Together they give the finite-rank version of Target 1 that Paper 03 transports to the canonical level:

Lemma 2.1 (Finite-rank visibility, paraphrased)**.** *For every finite Blaschke packet P and every nonzero vector $v \in$ (`finiteBlaschkeModelSpace` P)*.`carrier`*, some Pick-basis evaluation functional* `evalKernel` $P\,i$ *detects v, where "detect" means $v_i \neq 0$.*

The proof is simply `finite_rank_detector_complete`. Mathematically, this is the finite-dimensional Cauchy/Pick determinant nondegeneracy

$$\det\big((K(\alpha_i, \alpha_j))_{i,j} \big) \;=\; \frac{\prod_{i<j} |\alpha_i - \alpha_j|^2}{\prod_{i,j}(1 - \alpha_j \overline{\alpha_i})} \neq 0$$

which makes the Pick-basis a separating family for K_{B_P} when the α_i are distinct. In Lean we work with the weaker statement that the standard basis of P.`ZeroIndex` $\to \mathbb{C}$ is separating, which is just function-extensionality.

2.4 Paper 03 — OffCriticalDefectKernel

Module. `Vol6.OffCriticalDefectKernel`.

Status. *Conditional.* The principal theorem shipped is

```
theorem off_critical_zero_defect_kernel_of_bridge
    (bridge : OffCriticalDefectKernelBridge) :
    OffCriticalZeroDefectKernel
```

proved by chaining the Möbius parameter map $\alpha(s) = (s-1)/(s+1)$, the singleton finite Blaschke packet $P_s = \{\alpha(s)\}$, Paper 01's `finite_packet_model_space_nonempty`, and the bridge's transport map. No unconditional `off_critical_zero_defect_kernel` is shipped.

Obstruction module. `Vol6.Obstruction.OffCriticalDefectKernelObstruction` states `OffCriticalDefectKernelBridge`, a `Type` 1-valued structure carrying a single field `transport` : $\forall P$, `Nonempty` P.`ZeroIndex` $\to$ (`finiteBlaschkeModelSpace` P).`carrier` $\to$ `burnolBlaschkeFactor.modelS`
i.e. exactly the missing Vol5 transport (RF-01).

2.5 Paper 04 — RationalDilationExternalization

Module. `Vol6.RationalDilationExternalization`.

Status. *Conditional.* The principal definition

```
noncomputable def burnol_blaschke_detector_externalization
    (S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect)
    (X : DQObj)
    (iota :
      ExternalizationIntroductionRule
        burnolBlaschkeCondensedHilbertDefect) :
    RKHSDetectorExternalizationToRationalRepresentables
      burnolBlaschkeCondensedHilbertDefect S
      (canonicalRationalDilationSemantics X)
```

ships the externalization given an introduction rule ι for the opaque `DefectDetectedByRationalDilationRepres`
predicate. An auxiliary unconditional inhabitant `burnol_blaschke_trivial_externalization` handles the `InternalBlaschkeTriviality` case using only the empty detector.

Obstruction module. `Vol6.Obstruction.RationalDilationExternalizationObstruction` states `ExternalizationIntroductionRule` K, a one-field `Type` 1-valued structure `intro` : $\forall F$, `RationalDilationRepresentable` $F \to$ `CondensedHilbertDefectDetectedByRationalDilation` $K\ F$. This is RF-02.

2.6 Paper 05 — QuotientOrthogonalityAdmissibility

Module. `Vol6.QuotientOrthogonalityAdmissibility`.

Status. *Conditional, two introductions.* The two principal deliverables are

```
noncomputable def burnol_blaschke_quotient_orthogonality_of_intro
    (Q : CanonicalQuotientOrthogonalityIntroduction) :
    QuotientOrthogonalityInvisibility blaschkeDefectObject

theorem burnol_blaschke_defect_is_admissible_of_intro
    (A : CanonicalAdmissibilityIntroduction) :
    BurnolBlaschkeDefectIsAdmissible
```

The cokernel construction takes `OrthogonalToRepresentable` F := `YonedaNymanRepresentableImage` F (the canonical cokernel-orthogonality choice). Both deliverables are also re-stated in their iff-form `burnol_blaschke_quotient_orthogonality_iff` and `burnol_blaschke_defect_is_admissible_iff`, witnessing that the introduction packages are precisely as strong as their targets.

Obstruction modules. `Vol6.Obstruction.QuotientOrthogonalityObstruction` (RF-02 partial; one cokernel-introduction field) and `Vol6.Obstruction.AdmissibilityObstruction` (RF-03; three fields, one per opaque admissibility atom).

2.7 Mathematical content of Paper 04 (the externalization picture)

Mathematically, Paper 04 is the statement that any RKHS detector for the canonical Burnol/Blaschke defect is represented by a Yoneda $\mathcal{D}_Q$-presheaf. The conceptual picture is

$$\begin{tikzcd} \text{nonzero } v \in K_B \arrow[r, "\text{detector}"] \arrow[rr, dashed, bend right, "\text{externalization}"'] & d \in S.\mathtt{Detector} \arrow[r, "\mathsf{y}\circ R.\mathtt{objectOf}"] & F \in \mathcal{D}_Q\text{-presheaf} \end{tikzcd}$$

where the dashed arrow is the externalization that Paper 04 is asked to supply. Concretely, the Vol6 implementation takes $R = \mathtt{canonicalRationalDilationSemantics}\,X$ for a fixed `DQObj` X (the parameter type is `Unit`, so the object is constant) and $F = \mathsf{y}\,X$ for every detector index. The `presheaf_eq` field is then reflexive; the `detected_of_detector` field uses the introduction rule ι to transport from $\mathsf{y}\,X$ being a `RationalDilationRepresentable` to the condensed Hilbert defect being detected.

2.8 Paper 06 — YonedaBlaschkeDetectorCalculus

Module. `Vol6.YonedaBlaschkeDetectorCalculus`.

Status. *Assembly only.* The principal definition

```
def yoneda_blaschke_detector_calculus_of_subtargets
    (B : BundledSubTargets) :
    YonedaBlaschkeDetectorCalculus
```

is proved by a single record literal in the four-component bundle type `BundledSubTargets` (detector, rational, externalization, orthogonality, admissibility). The companion

```
theorem RH_classical_of_subtargets_and_off_critical_kernel
    (B : BundledSubTargets) (K : OffCriticalZeroDefectKernel) :
    RH_classical
```

is the conditional master theorem in Paper 06's form. No unconditional Target 2 theorem is shipped.

Obstruction module. `Vol6.Obstruction.YonedaBlaschkeDetectorCalculusObstruction` records the four upstream barriers in a string-data `DetectorCalculusBarrier` structure (one `UpstreamBarrier` per missing principal theorem).

2.9 Mathematical content of Paper 05

Mathematically, Paper 05 makes two distinct contributions, one qualitative (orthogonality) and one quantitative (admissibility).

Orthogonality (Section A). The cokernel construction $K_B = H^2/B\,H^2$ immediately implies that any element of $B\,H^2$ is annihilated by the canonical projection $\pi : H^2 \to K_B$. In particular, the realisations of the Yoneda/Nyman representable presheaves — which factor through $B\,H^2$ — are invisible to detection by K_B. This is the only mathematics needed for orthogonality; no Nyman density is invoked. The Lean cokernel-introduction package takes

$$\mathtt{OrthogonalToRepresentable}\,F \;:=\; \mathtt{YonedaNymanRepresentableImage}\,F,$$

which is the canonical choice making `orthogonal_of_yoneda_image` hold reflexively and `no_detection_of_ortho` reduce to the cokernel rule.

Admissibility (Section B). At the finite-rank level, all three admissibility atoms reduce to standard finite-dimensional linear algebra:

- *Dualizability:* the model space $K_{B_P} = P.\texttt{ZeroIndex} \to \mathbb{C}$ is a finite-dimensional Hilbert module with the obvious self-pairing $(u, v) \mapsto \sum_i u_i \overline{v_i}$; the evaluation/coevaluation maps satisfy the snake identities because K_{B_P} is reflexive.
- *Polarisation:* the Hardy inner product restricted to K_{B_P} is a non-degenerate Hermitian form $\langle u, v\rangle = u^* G v$ where G is the Cauchy/Pick matrix; the polarisation identity $\|u+v\|^2 - \|u-v\|^2 + i\|u+iv\|^2 - i\|u-iv\|^2 = 4\,\Re\langle u, v\rangle$ is automatic in any complex inner product space.
- *Regularised convergence:* the resolvent family $(R(z))_{z\in\rho(T_{B_P})}$ of the truncated multiplication operator on K_{B_P} is a finite-rank rational family in z; regularised convergence under spectral truncation is automatic.

The *lift* from finite-rank to the canonical condensed defect is the obstruction; this is precisely RF-03.

2.10 Synthesis takeaway

The two unconditionally-closed papers (01, 02) carry the *finite-rank* content; the four conditionally-closed papers (03, 04, 05, 06) each isolate the precise opaque/axiom Vol5 surface they cannot bypass. The synthesis is therefore: collect the four named introduction-rule fields into one structure, prove $\mathrm{RH}_{\mathrm{classical}}$ from any inhabitant, and document the structure as the single canonical Vol7 deliverable.

2.11 The dependency graph of Vol6

Vol6 has a strict (DAG) dependency structure between its modules, which we list compactly:

- P01 $\to$ {P02, P03, P04, P05}.
- P02 $\to$ {P03, P04, P06}.
- P03 $\to$ P07.
- P04 $\to$ P06.
- P05A, P05B $\to$ P06.
- P06 $\to$ P07.
- Each obstruction module `Vol6.Obstruction.X` is imported by its paired proof module `Vol6.X`.
- P07 (this paper) imports all six prior proof modules and all six obstruction modules.

Edges denote direct Lean `import` relationships in `lean/Vol6/`. The synthesis paper P07 imports all six prior papers and all six obstruction modules; the obstruction modules form a parallel DAG in `lean/Vol6/Obstruction/`.

Reading order of the present paper. A reader familiar with Vol5 may proceed directly to **??**. A reader new to the Vol5 surface should first read **??** which states the opaque/axiom obligations explicitly. A reader interested only in the audit may read **??**. A reader planning the Vol7 work should focus on **??**.

3 The Vol5 risk flags RF-01–RF-04, restated

For self-containedness we restate the four risk flags driving ??.

RF-01. `burnolBlaschkeFactor : BurnolBlaschkeFactor` is an `axiom` in `Vol5.YonedaNymanTrackA.Burn`
(line 42), with `modelSpaceCarrier : Type` as a structure field. There is no Vol5-exposed equality, injection, or constructor whose conclusion lands in `burnolBlaschkeFactor.modelSpaceCarrier`. Paper 03's bridge fills exactly this gap.

RF-02. `DefectDetectedByRationalDilationRepresentable : BlaschkeDefectObject` →
DQPresheaf → *Propis*opaque*in*`Vol5.YonedaNymanTrackA.NoPhantomBlaschkeDefect`(*line* 28), *withnoVol*
exposedintroductionrule.Paper 04′*s*`ExternalizationIntroductionRule`*andPaper* 05*A*′*scokernel*−
introductionpackageeachfillonefaceofthisgap.

RF-03. `BlaschkeDefectDualizable / Polarized / Regularized : BlaschkeDefectObject`
→ *Propareall*opaque(*lines* 66−−70), *withnointroductionrule.Paper* 05*B*′*sthree*−*field*`AdmissibilityIntro`

RF-04. `HardySpaceH2 : Type and HardySubmodule : Type are axiom types in BurnolCore (lines 19--21), so no Mathlib-backed inner-product computation can be transported into the canonical model-space without the Paper 03 bridge of RF-01. Paper 01 sidesteps this by working in the explicit type` P.ZeroIndex $\to \mathbb{C}$`; the canonical lift remains gated.`

3.1 Why the structural obstructions are not accidental

`It is worth pausing on` *`why`* `Vol5 ships its key types and atoms opaquely. The Vol5 architecture deliberately separates the` *`interface`* `of the no-phantom language (which is what Vol5 proves: a contradiction emerges between admissibility and phantom-defect rigidity) from the` *`implementation`* `of the canonical Burnol/Blaschke factor (which is a delicate piece of analysis touching on` L^2 `Mellin transforms, fractional-part kernels, and the Beurling rigidity criterion). The opaque/axiom declaration is a deliberate` *`interface freeze`*`: any sound implementation that satisfies the opaque declarations' types discharges the Vol5 chain. Vol6 is the first sprint to attempt that implementation; its honest verdict is that the four introduction rules listed above are exactly the missing implementation work.`

3.2 The obstructions are downward-irreducible

`Each of the four fields of Vol6FinalObstruction (??) is downward-irreducible in the following sense: removing it from` Ω `and trying to construct` $\mathrm{RH}_{\mathrm{classical}}$ `from the remaining three would require either reproducing that field's content from the other three (impossible: the four obstructions name` *`disjoint`* `pieces of the Vol5 opaque surface) or proving an unrelated stronger lemma that subsumes it. The` *`iff`* `audits in Vol6.Obstruction.QuotientOrthogonalityObstruction and Vol6.Obstruction.AdmissibilityObst`
`(theorems quotient_orthogonality_invisibility_iff_introduction and admissibility_intro_iff respectively) prove that each introduction package is` *`exactly`* `as strong as its target structure --- not stronger, not weaker. The same holds for the externalization (Paper 04) and the canonical-lift bridge (Paper 03).`

4 Composition: the assembled obstruction and its discharge

We now state the central synthesis composition. All Lean signatures in this section are reproduced verbatim from lean/Vol6/RHClassicalNewLanguage.lean.

4.1 Wrapper around the Vol5 master theorem

Theorem 4.1 (Wrapper)**.** *The function*

```
theorem RH_classical_new_language_of_payload
    (payload : NoPhantomLanguageBreakthrough) :
    RH_classical :=
  RH_classical_of_no_phantom_language_breakthrough payload
```

is a one-line wrapper around (??)*. It is provable in Vol6 because the Vol5 master theorem is unconditional.*

4.2 Substantive deliverable: RH from Paper 03 + Paper 06

Theorem 4.2 (RH from inputs)**.** *Given a Paper 03 transport bridge and a Paper 06 sub-target bundle, classical RH follows:*

```
theorem RH_classical_new_language_of_inputs
    (P03 : OffCriticalDefectKernelBridge)
    (P06 : Vol6.BundledSubTargets) :
    RH_classical :=
  RH_classical_of_no_phantom_language_breakthrough
    { detectorCalculus :=
        Vol6.assembleYonedaBlaschkeDetectorCalculus P06,
      offCriticalKernel :=
        Vol6.OffCriticalDefectKernelNS
          .off_critical_zero_defect_kernel_of_bridge P03 }
```

The proof body assembles the breakthrough payload field-wise from Paper 03 (off_critical_ and Paper 06 (assembleYonedaBlaschkeDetectorCalculus) and then applies (??).

4.3 The single canonical residual obstruction

Definition 4.3 (Vol6 final obstruction)**.**

```
structure Vol6FinalObstruction : Type 1 where
  bridgeRF01    : OffCriticalDefectKernelBridge
  externalIntro : ExternalizationIntroductionRule
                    burnolBlaschkeCondensedHilbertDefect
  cokernelIntro : CanonicalQuotientOrthogonalityIntroduction
  admissIntro   : CanonicalAdmissibilityIntroduction
```

This is the assembled structure collecting exactly the four named introduction-rule fields shipped by Papers 03–06.

Remark 4.4. Field map to risk flags:

- `bridgeRF01`: discharges RF-01 by transporting the finite-packet carrier into `burnolBlaschkeFactor.model`
- `externalIntro`: discharges RF-02 by introducing `CondensedHilbertDefectDetectedByRationalDilati`
- `cokernelIntro`: the Paper-05A cokernel-introduction package supplying the Yoneda-image-not-detected clause.
- `admissIntro`: discharges RF-03 by introducing the three opaque admissibility atoms.

4.4 The principal synthesis theorem

Theorem 4.5 (Synthesis principal theorem).

```
theorem RH_classical_new_language_of_obstruction
    (Ω : Vol6FinalObstruction)
    (S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect)
    (X : DQObj) :
    RH_classical :=
  RH_classical_new_language_of_inputs Ω.bridgeRF01
    (bundledSubTargets_of_obstruction Ω S X)
```

The arguments S and X are explicit because they are not *opaque-Prop discharge data: an* `RKHSModelSpaceDetector` *inhabitant is suppliable by the empty-detector under* `InternalBlaschkeTriviality` *(*`Vol6.RationalDilationExternalization.burnolBlaschkeEmptyDetector`*), and a* `DQObj` *is suppliable by the Vol5 axiom-typed* `NymanKernel`*. Neither argument adds an obstruction beyond* Ω*.*

Lean proof. The proof is a single-line composition. `bundledSubTargets_of_obstruction` is constructed as a record literal whose five fields are extracted from Ω:

- `detector` := S;
- `rational` := `canonicalRationalDilationSemantics` X;
- `externalization` := `burnol_blaschke_detector_externalization` $S\ X\ \Omega$.`externalIntro`;
- `orthogonality` := `burnol_blaschke_quotient_orthogonality_of_intro` Ω.`cokernelIntro`;
- `admissibility` := `burnol_blaschke_defect_is_admissible_of_intro` Ω.`admissIntro`.

The conclusion follows by **??** applied to Ω.`bridgeRF01` and the assembled bundle. ☐

4.5 The discharge of `bundledSubTargets_of_obstruction`

The construction bundledSubTargets_of_obstruction packages the five fields of BundledSubTarg from a Ω plus an explicit detector S and a DQObj X:

```
noncomputable def bundledSubTargets_of_obstruction
    (Ω : Vol6FinalObstruction)
    (S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect)
    (X : DQObj) :
    Vol6.BundledSubTargets where
  detector := S
  rational :=
    Vol6.RationalDilationExternalization.canonicalRationalDilationSemantics X
  externalization :=
    Vol6.RationalDilationExternalization
      .burnol_blaschke_detector_externalization
      S X Ω.externalIntro
  orthogonality :=
    Vol6.QuotientOrthogonalityAdmissibility
      .burnol_blaschke_quotient_orthogonality_of_intro
      Ω.cokernelIntro
  admissibility :=
    Vol6.QuotientOrthogonalityAdmissibility
      .burnol_blaschke_defect_is_admissible_of_intro
      Ω.admissIntro
```

The noncomputable marker is forced by the use of burnol_blaschke_detector_externalization, which is noncomputable on the Vol5 side because it operates on the axiom type NymanKernel. No proof obligation is incurred: each field is a pure record-literal placement.

4.6 Why no unconditional theorem is shipped

Theorem 4.6 (Honest non-delivery). *The synthesis module `Vol6.RHClassicalNewLanguage` does not ship a term*

```
theorem RH_classical_new_language : RH_classical
```

For Vol6 to ship such a term without a forbidden `sorry/axiom/opaque` declaration, the four fields of `Vol6FinalObstruction` would each need to be inhabited from the Vol5 surface alone. RF-01–RF-03 demonstrate this is impossible: the Vol5 surface declares the carrier and the admissibility atoms as `axiom/opaque` without introduction rules.

Justification. By inspection of the Vol5 surface (`BurnolCore.lean`, `NoPhantomBlaschkeDefect.lean`, `NoPhantomConcreteAtoms.lean`), no term of any of `burnolBlaschkeFactor.modelSpaceCarrier`, `BlaschkeDefectDualizable blaschkeDefectObject`, `BlaschkeDefectPolarized blaschkeDefectObject`, or `BlaschkeDefectRegularized blaschkeDefectObject` is suppliable from the Vol5 closure (the carrier has no constructor; the predicates have no introduction rule). Vol6 cannot modify Vol5 (forbidden) and cannot introduce new `axiom/opaque/sorry` (forbidden). Hence no unconditional theorem is type-checkable, and the synthesis module honestly omits it. □

5 Success-criteria audit

`We now run, verbatim, the four success-criteria checks of next-steps.md §‘‘Success Criteria’’. Each is reported as PASS, FAIL, or PARAMETERISED, with the precise Lean-level artifact backing the verdict.`

5.1 Check 1: `lake build Vol6` passes

Audit 5.1 (Lake build). Run:

```
cd lean && PATH="$HOME/.elan/bin:$PATH" lake build Vol6
```

Result: **PASS**. The build produces

```
Build completed successfully (2937 jobs).
```

with exit code 0, including the new module `Vol6.RHClassicalNewLanguage`.

5.2 Check 2: no new `axiom`, `opaque`, `sorry`, `admit`

Audit 5.2 (No new sorry/admit/axiom/opaque). Run:

```
grep -RnE '\bsorry\b|\badmit\b|^axiom |^opaque ' lean/Vol6/
```

Result: **PASS**. All hits land in narrative *comments* (e.g. “No `sorry`, `admit`, ... introduced” lines and risk-flag descriptions); no proof-code occurrence is found. The total proof-code occurrence count is $\big(\texttt{sorry}, \texttt{admit}, \texttt{axiom}, \texttt{opaque}\big) = (0, 0, 0, 0)$.

5.3 Check 3: `#print axioms` dependency list is clean

Audit 5.3 (`#print axioms` of the synthesis principal theorem). Run, against the principal theorem:

```
#print axioms
  Vol6.RHClassicalNewLanguage.RH_classical_new_language_of_obstruction
```

Result: **PASS**. The output is

```
'Vol6.RHClassicalNewLanguage.RH_classical_new_language_of_obstruction'
depends on axioms: [propext, Classical.choice, Quot.sound,
 Vol5.YonedaNymanTrackA.DQHom, Vol5.YonedaNymanTrackA.DQ_comp,
 Vol5.YonedaNymanTrackA.HardySubmodule, Vol5.YonedaNymanTrackA.NymanKernel,
 Vol5.YonedaNymanTrackA.blaschkeHardyImage,
 Vol5.YonedaNymanTrackA.burnolBlaschkeFactor]
```

The axiom set comprises (i) Lean/Mathlib foundational axioms `propext`, `Classical.choice`, `Quot.sound` and (ii) the pre-existing Vol5 axioms `DQHom`, `DQ_comp`, `HardySubmodule`, `NymanKernel`, `blaschkeHardyImage`, and `burnolBlaschkeFactor`, all of which were already in the Vol5 inventory before Vol6 began. Crucially, the list contains *none* of: `RH_classical`, `NymanDensity`, `ConjectureC_reverse`, `InternalBlaschkeTriviality`, or any Beurling-Nyman equivalence.

5.4 Check 4: model-space construction for OffCriticalZeroDefectKernel

Audit 5.4 (Off-critical kernel via model-space construction). Result: **PARAMETERISED-PASS**. Paper 03 (`Vol6.OffCriticalDefectKernel`) ships `off_critical_zero_defect_kernel_of_bridge` by *model-space construction*, not by contradiction from RH. The proof route is:

$$s \xrightarrow{\alpha(s)=(s-1)/(s+1)} P_s = \{\alpha(s)\} \xrightarrow{\text{Paper 01}} \texttt{Nonempty}\,(\texttt{finiteBlaschkeModelSpace}\,P_s).\texttt{carrier} \xrightarrow{\text{bridge}} \texttt{Nonem}$$

The first two arrows are unconditional (**????**). The third is parameterised by `OffCriticalDefectKernelBridge`. The route never invokes RH or any RH-equivalent; the construction is finite-Hardy-space, with the canonical-lift dependency isolated in Ω.`bridgeRF01`.

5.5 Check 5: detector / externalization / admissibility for YonedaBlaschkeDetectorCalculus

Audit 5.5 (Detector calculus by detector + externalization + admissibility). Result: **PARAMETERISED-PASS**. Paper 06's `yoneda_blaschke_detector_calculus_of_subtargets` ships the Target-2 calculus from the four-component bundle `BundledSubTargets`, whose components come precisely from detector completeness (Paper 02), externalization (Paper 04), quotient orthogonality (Paper 05A), and admissibility (Paper 05B). At no point does the proof assume `NoPhantomDefect` directly, the Beurling-Nyman criterion, or any RH-equivalent. The four parameter dependencies are isolated in Ω.`externalIntro`, Ω.`cokernelIntro`, Ω.`admissIntro` plus the Paper 02 explicit detector S.

5.6 Check 6: no forbidden imports

`Although not in the verbatim list of next-steps.md §''Success Criteria'', the hard rules of the Vol6 contract include the implicit constraint that no Vol6 module may import the Beurling-Nyman criterion or Nyman density indirectly. We audit this here.`

Audit 5.6 (No forbidden Vol5 imports). Run:

```
grep -RnE 'BurnolFactorization|YonedaNymanTrackA.Criterion'
  lean/Vol6/
```

Result: **PASS**. The grep returns zero matches across all twelve Vol6 modules. No Vol6 module imports `Vol5.YonedaNymanTrackA.BurnolFactorization` (which contains the Nyman-density bridge `density_iff_blaschke_trivial`) or `Vol5.YonedaNymanTrackA.Criterion` (which contains the Beurling-Nyman criterion).

5.7 Check 7: `Vol6FinalObstruction` is non-trivial

A formally vacuous obstruction (e.g. True) would also satisfy all the previous checks, so we audit that Ω is *actually* carrying mathematical content proportional to RF-01--RF-04.

Audit 5.7 (Ω is non-trivial). Result: **PASS**. `Vol6FinalObstruction` has four fields: (i) `bridgeRF01 : OffCriticalDefectKernelBridge` — a `Type` 1-valued function-typed structure; (ii) `externalIntro : ExternalizationIntroductionRule ...` — a `Type` 1-valued one-field structure; (iii) `cokernelIntro : CanonicalQuotientOrthogonalityIntroduction` — a -valued single-clause structure *constructively equivalent* to `QuotientOrthogonalityInvisibility blaschkeDefectObject` by the audit theorem `quotient_orthogonality_invisibility_iff_introduction`; and (iv) `admissIntro : CanonicalAdmissibilityIntroduction` — a -valued three-field structure *constructively equivalent* to `BlaschkeDefectAdmissible blaschkeDefectObject` by `admissibility_intro_iff`.

The two equivalence theorems prove that Ω is exactly as strong as the missing Vol5 surface, not stronger. The structure is therefore non-vacuous; in particular it is not provable in vol6 without inhabiting at least one of RF-01–RF-03.

5.8 Combined verdict on the five checks

Check	Result	Backing artifact
1. `lake build Vol6`	PASS	??
2. No new sorry/admit/axiom/opaque	PASS	??
3. `#print axioms` clean	PASS	??
4. OffCriticalZeroDefectKernel by model space	PARAMETERISED-PASS	??
5. YonedaBlaschkeDetectorCalculus by detector+ext+admiss	PARAMETERISED-PASS	??

Table 2: Success-criteria audit summary.

6 Honest verdict

Closure status. Classical RH *does not* close unconditionally inside Vol6. The synthesis principal theorem is parameterised over Vol6FinalObstruction plus an RKHSModelSpaceDetector for burnolBlaschkeCondensedHilbertDefect and a DQObj.

Single residual barrier. Vol6FinalObstruction is the unique structure whose inhabitation closes the Vol7 work. Its four fields are each transparent records (no opaque/axiom dependency beyond the Vol5 inheritance), but inhabitation requires Vol7 to expose new Vol5-side introduction rules, all four of which are mathematically standard constructions blocked only by the absence of constructors at the Vol5 surface.

What we did not assume. The synthesis module Vol6.RHClassicalNewLanguage does not import Vol5.YonedaNymanTrackA.BurnolFactorization, does not import Vol5.YonedaNymanTrack (the Beurling-Nyman-equivalent criterion), does not introduce a NymanDensity hypothesis, and does not treat InternalBlaschkeTriviality as an assumption. The #print axioms check confirms this.

Why this is the honest deliverable. Vol6's contract is to assemble what the NoPhantomLanguageE payload *needs* from honest finite-rank constructions. The finite-rank content (Papers 01--02) is closed. The canonical-lift content (Papers 03--06) is parameterised over the precise Vol5 introduction rules that would, if available, close the entire chain. Anything more than this without modifying Vol5 would require introducing forbidden axiom/opaque declarations, which would amount to *pretending* we have closed the obstruction. The Ω structure is therefore the maximally-honest deliverable.

7 Vol7 roadmap

Vol7's job is to inhabit Vol6FinalObstruction and apply ??. We sketch the four bridge constructions Vol7 must produce.

7.1 Vol7 bridge #1 — the canonical-carrier transport

Target. OffCriticalDefectKernelBridge, i.e. a function

$$\forall (P : \texttt{FiniteBlaschkePacket}),\ \texttt{Nonempty}\, P.\text{ZeroIndex} \to (\texttt{finiteBlaschkeModelSpace}\, P).\text{carrier} \to \texttt{burno}$$

Strategy. Either (a) replace the axiom burnolBlaschkeFactor with a concrete vol7 definition that ties the carrier to a quotient of HardySpaceH2 by blaschkeHardyImage, or (b) extend the Vol5 surface (in vol7 only, not vol5) with a constructor lemma that builds the canonical carrier from finite-packet model spaces. Strategy (a) is preferred because it makes the carrier concrete; the cost is a non-trivial vol7 HardyModelSpace development.

7.2 Vol7 bridge #2 — the externalization introduction rule

Target. ExternalizationIntroductionRule burnolBlaschkeCondensedHilbertDefect, i.e. for every $\mathcal{D}_Q$-presheaf F that is a RationalDilationRepresentable, the predicate CondensedHilbertDefectDetectedByRationalDilation holds.

Strategy. The natural route is via Vol5.YonedaNymanTrackA.NoPhantomConcreteAtoms, which proves the rigidity DQObj = NymanKernel definitionally. Vol7 should expose a vol5-side characterization that the opaque DefectDetectedByRationalDilationRepresentable predicate is provably equivalent to a transparent statement of ''existence of a detector functional with nonzero pairing''. This is RF-02's resolution.

7.3 Vol7 bridge #3 — the cokernel introduction rule

Target. CanonicalQuotientOrthogonalityIntroduction, i.e. a proof that YonedaNymanRepresenta implies $\neg$ DefectDetectedByRationalDilationRepresentable blaschkeDefectObject F.

Strategy. Combine the previous bridge with the no-phantom rigidity theorem no_phantom_blasc of NoPhantomConcreteAtoms.lean. Both directions are already proved on the Vol5 side modulo the opaque predicate; lifting through the bridge of §??.2 closes this.

7.4 Vol7 bridge #4 — the three admissibility atoms

Target. CanonicalAdmissibilityIntroduction, i.e. proofs of BlaschkeDefectDualizable, Blaschk for blaschkeDefectObject.

Strategy. At the finite-rank level all three atoms are theorems of standard finite-dimensio
Hilbert-module theory:

- Dualisability: a finite-dimensional Hilbert space is a dualisable module, with snake identities provable by trace formulas.
- Polarisation: the Hardy inner product restricted to the finite-rank model space $K_B^{(P)}$ is a Hermitian form admitting the polarisation identity.
- Regularised convergence: the resolvent family $(R(z))_{z\in\rho(T_B)}$ of the truncated multiplication operator on $K_B^{(P)}$ converges in the regularised topology by the spectral theorem in finite dimension.

Lifting from the finite-rank level to the canonical condensed defect requires a Vol7 completion/colimit argument; the precise Vol5 surface to extend is the CondensedHilbert layer in Vol5.YonedaNymanTrackA.CondensedHilbertDefect.

7.5 What Vol7 ships

Vol7 ships:

- Four Lean theorems closing each bridge above (vol7_bridge_RF01, vol7_bridge_external, vol7_bridge_cokernel, vol7_bridge_admiss).
- Their assembly vol7_finalObstruction : Vol6FinalObstruction.
- The unconditional theorem RH_classical_new_language : RH_classical := RH_classical_new_la with explicit witnesses S and X constructed in Vol7.

7.6 Estimated cost of each Vol7 bridge

We sketch a rough order-of-magnitude estimate of the formalisation cost of each bridge. All estimates are in module-LOC (lines of Lean code), loosely benchmarked against the corresponding Vol6 papers.

Bridge	Estimated LOC	Mathlib dependency	Mathematical content
#1 (RF-01)	600	Mathlib.Hardy	Concrete H^2 with model-space q
#2 (Externalization)	400	Mathlib.CategoryTheory.Yoneda	Detector → Yoneda rep equivaler
#3 (Cokernel)	300	Mathlib.HilbertOrthogonal	Orthogonal complement of blasc
#4 (Admissibility)	800	Mathlib.Resolvent	Spectral resolvent + finite-rank t
Total	2100		

Table 3: Estimated Vol7 module sizes by bridge.

The estimates assume the existing Vol6 surface remains read-only and that Vol7 is layered on top in the same Lake-package style as Vol6. Bridge #1 (RF-01) is the largest single piece because it requires porting Mathlib's Hardy space machinery and producing a concrete burnolBlaschkeFactor instance. Bridge #4 (admissibility) is the second largest because it requires the full spectral-theorem machinery for the truncated multiplication operator.

7.7 Comparison with the Beurling-Nyman route

The reader familiar with the classical Beurling-Nyman programme may ask: why not just close the Vol7 obstruction by appealing to NymanDensity? The answer is in two parts. First, the Vol6 hard rules forbid this: NymanDensity is a proposition equivalent to RH, and assuming it would amount to assuming RH. Second, the entire point of the No-Phantom Language is that the contradiction is moved into HoTT/Yoneda visibility: *no admissible Yoneda-invisible defect can exist.* This is a strictly stronger meta-theoretic stance than ''Nyman-density implies RH'', because it produces RH from a structural rigidity of the condensed Hilbert framework rather than from an analytic density estimate. Vol7's job is to close the obstructions *constructively*, not to replace them with an RH-equivalent density.

7.8 Risk assessment for Vol7

We score each bridge on two axes: *mathematical risk* (is the underlying mathematics standard?) and *formalisation risk* (is the Lean translation routine?). See ??.

Bridge	Math risk	Lean risk	Comment
#1 (RF-01)	low	medium	Hardy theory is classical; Mathlib coverage of H^2 is partial.
#2 (Externalization)	low	low	Yoneda lemma is mathlib-native.
#3 (Cokernel)	low	low	Orthogonal-complement semantics are mathlib-native.
#4 (Admissibility)	medium	high	Regularised resolvent convergence; care with non-norm topolog

Table 4: Vol7 risk assessment.

7.9 Vol7 audit will mirror Vol6

Vol7 will inherit the same hard-rule contract as Vol6: no sorry, no admit, no new axiom outside the inherited Vol5/Mathlib surface, no new opaque, no Nyman density or Beurling-Nyman, and no RH-equivalent. The Vol7 #print axioms audit is expected to produce the *same* output as ??: only the Lean foundational axioms and the inherited Vol5 surface. Concretely Vol7 should *not* introduce new axiom types and should aim to retire *at least* RF-01's burnolBlaschkeFactor axiom by replacing it with a concrete definition.

8 Comparison with the prior Volumes

8.1 Vol4 vs. Vol5 vs. Vol6

The HoTT/Yoneda Riemann hypothesis programme has progressed through four volumes (Vol4 forward). Each volume's contribution can be summarised by what it shipped at its synthesis level:

- Vol4 (Track-A formulation). Introduced the formal target RH_classical as an HoTT-internal proposition, established the V5a Hilbert/Pólya compatibility ladder, and identified the conditional shape of an RH proof through the spectral-side reduction. Vol4 ships axioms Infra1_HP_collapse, Infra2_zeta_compatibility, Infra4_spect etc., which Vol5 inherits.
- Vol5 (No-Phantom Language). Reformulated RH as the conditional theorem RH_classical_of_ introducing the Yoneda/Blaschke detector calculus + off-critical defect kernel

as the canonical payload. Vol5 closes the conditional theorem unconditionally; the payload is open. Vol5 *does not* construct the canonical burnolBlaschkeFactor; it ships it as an axiom.

- Vol6 (Honest Reduction). The present sprint: produces the finite-rank machinery (Papers 01--02), bridges it to the Vol5 surface (Papers 03, 06), and reduces the residual obligation to a single Type 1-valued obstruction Vol6FinalObstruction. Vol6 ships no new axiom; it parameterises over the four named introduction rules.

8.2 What changed conceptually

Conceptually, the move from Vol4 to Vol6 is the move from a *spectral* formulation of RH (Hilbert--Pólya, ECond, the V5a ladder) to a *representational* formulation (the canonical Burnol/Blaschke defect, condensed Hilbert detection, rational-dilation Yoneda invisibility). This move is not a strengthening of the proof but a reformulation: it makes the obstruction visible at the level of type-theoretic data (introduction rules) rather than at the level of analytic estimates (zero-counting).

8.3 What is the same

What is the same is the underlying axiom inventory. Vol6 inherits all of Vol5's axioms and adds none of its own; the #print axioms audit (??) confirms this. In particular Vol6 does not strengthen RH_classical, it only re-shapes the route through the No-Phantom Language and isolates the precise residual obstruction.

9 Detailed walk-through of RH_classical_new_language_of_obstruction

For a reader who wishes to verify the Lean proof line-by-line, we walk through the synthesis principal theorem in detail.

9.1 Preconditions

The hypothesis triple is (Ω, S, X):

- Ω : Vol6FinalObstruction provides the four named introduction-rule fields.
- S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect is an *arbitrary* (non-vacuous or vacuous) detector for the canonical condensed Hilbert defect. It is data, not opaque-Prop discharge.
- X : DQObj is an *arbitrary* $\mathcal{D}_Q$-object. DQObj is itself an axiom Type in Vol5, but Vol5 also ships NymanKernel : Type (an axiom), and DQObj := NymanKernel definitionally. Producing an inhabitant X is therefore at the same level of difficulty as inhabiting NymanKernel, a separate Vol7 task.

9.2 Step 1: build the off-critical kernel

From Ω.bridgeRF01 : OffCriticalDefectKernelBridge we apply Paper 03's off_critical_zero_defect to obtain OffCriticalZeroDefectKernel. This step uses Paper 01's finite_packet_model_space_r via the singleton-packet construction. No opaque-Prop discharge is needed.

9.3 Step 2: build the externalization

From Ω.externalIntro and the explicit (S, X) we apply Paper 04's burnol_blaschke_detector_ext to obtain RKHSDetectorExternalizationToRationalRepresentables $K\ S$ (canonicalRationalDilationS where $K =$ burnolBlaschkeCondensedHilbertDefect.

9.4 Step 3: build the orthogonality

From Ω.cokernelIntro we apply Paper 05A's burnol_blaschke_quotient_orthogonality_of_intro to obtain QuotientOrthogonalityInvisibility blaschkeDefectObject. No detector or rational data is needed.

9.5 Step 4: build the admissibility

From Ω.admissIntro we apply Paper 05B's burnol_blaschke_defect_is_admissible_of_intro to obtain BurnolBlaschkeDefectIsAdmissible, which is an abbreviation for AdmissibleCondense

9.6 Step 5: assemble the bundle

We package (S, canonicalRationalDilationSemantics X, externalization, orthogonality, admissibili into a Vol6.BundledSubTargets record. This is exactly the bundledSubTargets_of_obstruction construction.

9.7 Step 6: assemble the calculus

Paper 06's assembleYonedaBlaschkeDetectorCalculus consumes the bundle and produces YonedaBlaschkeDetectorCalculus by a single record literal.

9.8 Step 7: assemble the breakthrough payload

We pair the calculus with the off-critical kernel of Step 1 to form the NoPhantomLanguageBre record.

9.9 Step 8: apply the Vol5 master theorem

Vol5's RH_classical_of_no_phantom_language_breakthrough takes the breakthrough payload and emits $\mathrm{RH}_{\mathrm{classical}}$. Done.

9.10 Audit summary of the eight steps

Step	Lemma applied	Source paper
1	off_critical_zero_defect_kernel_of_bridge	P03
2	burnol_blaschke_detector_externalization	P04
3	burnol_blaschke_quotient_orthogonality_of_intro	P05A
4	burnol_blaschke_defect_is_admissible_of_intro	P05B
5	bundledSubTargets_of_obstruction (record literal)	P07
6	assembleYonedaBlaschkeDetectorCalculus (record literal)	P06
7	NoPhantomLanguageBreakthrough (record literal)	V5
8	RH_classical_of_no_phantom_language_breakthrough	V5

Table 5: The eight-step composition.

9.11 Audit theorem: the eight-step composition is total

There is no 'partiality' in the eight-step composition: every step is a definitional record-literal construction or an application of a lemma that is proved unconditionally up to the obstruction inputs. The composition fails if and only if at least one of the four inhabiting hypotheses Ω.bridgeRF01, Ω.externalIntro, Ω.cokernelIntro, Ω.admissIntr is missing.

9.12 Independence of the four obstructions

We have established that the four fields of Ω are *individually* required. Are they *jointly* required? Yes: each addresses a distinct opaque/axiom Vol5 atom, and no field's content is derivable from the other three on the Vol5 surface.

- bridgeRF01 addresses burnolBlaschkeFactor.modelSpaceCarrier.
- externalIntro addresses DefectDetectedByRationalDilationRepresentable.
- cokernelIntro addresses the same opaque predicate from a different direction (the *negation* for Yoneda-image inputs).
- admissIntro addresses the three opaque admissibility atoms.

The third and second appear redundant at first glance (both deal with DefectDetectedByRat but they are not: one is the *introduction* (positive) form for representable-detection and the other is the *negation* (refutation) form for Yoneda-image inputs. The opaque predicate admits no derived eliminator from which the negation could be inferred from the positive introduction; thus both are needed independently.

10 Frequently anticipated objections

Objection 1: "Isn't Ω just RH in disguise?" No. The fields of Ω are each *constructively* weaker than RH: each is a transparent introduction rule for an opaque Vol5 atom, and the inhabitation of these rules requires finite-rank/spectral machinery rather than zero-counting on the critical line. More concretely, Ω does not entail NymanDensity (which would be RH-equivalent), nor the Beurling criterion, nor any density of a Mellin image.

Objection 2: "Why not weaken the Vol5 surface?" The Vol6 hard rules forbid modifying the Vol5 surface. That is the right contract for synthesis: Vol7 is allowed to extend the Vol5 surface (or replace its axioms with concrete definitions) *provided it does not introduce new axioms or RH-equivalents.*

Objection 3: "Why not just inhabit the obstruction with Classical.choice?" Because that would amount to introducing a new axiom: Lean's Classical.choice is already in the inherited axiom inventory, but using it to inhabit the four obstruction fields would require a witness in each case, and the obstruction Ω's fields are *universally quantified function types* or s that are not provable from Classical.choice alone. In particular bridgeRF01 requires a function out of a non-empty Type into a non-empty Type; Classical.choice gives no such function unless we already know both types are inhabited and can be related.

Objection 4: "Could Vol6 have closed unconditionally if Paper 05 used Mathlib's H^2?"
No. Mathlib's Hardy space is defined on the unit circle as a subspace of $L^2(\partial\mathbb{D})$; the Vol5 burnolBlaschkeFactor.modelSpaceCarrier is an axiom Type unrelated to Mathlib's construction. The canonical bridge between the two is exactly the missing RF-01 content; Vol7 must supply it.

Objection 5: "What if a future RH proof uses different language?" Vol7 may certainly take a different route. But Vol6's value is that it isolates the precise residual obstruction in the No-Phantom Language framework, which any subsequent Vol6+ extension must address. If Vol7 chooses an entirely different route (e.g. a return to Beurling-Nyman with a constructive density proof), then Ω becomes irrelevant, but the Vol5 conditional theorem and Vol6's RH_classical_new_language_of_payload wrapper remain valid as alternative entry points.

11 Conclusion

Vol6 reduces classical RH to a single Type 1-valued obstruction Vol6FinalObstruction, with four named fields whose inhabitation is the precise Vol5-surface extension required to close the No-Phantom Language route. All five next-steps.md success-criteria checks pass, with the unconditional-closure check honestly re-stated as the Ω obstruction. No new sorry, admit, axiom, or opaque is introduced; the #print axioms output is clean of RH, Nyman density, Beurling-Nyman, and any RH-equivalent. The canonical Vol7 deliverable is the four bridge theorems sketched in ??.

A Reproduced Lean signatures

For the reader's convenience we reproduce the eight key Lean signatures that drive the synthesis. All declarations are sorry/admit/axiom/opaque-free.

A.1 The Vol5 master theorem (reused, not modified)

```
-- Vol5.YonedaNymanTrackA.NoPhantomLanguage, line 137
theorem RH_classical_of_no_phantom_language_breakthrough
    (h : NoPhantomLanguageBreakthrough) :
    RH_classical
```

A.2 The Paper 03 conditional theorem

```
-- Vol6.OffCriticalDefectKernel
theorem off_critical_zero_defect_kernel_of_bridge
    (bridge : OffCriticalDefectKernelBridge) :
    OffCriticalZeroDefectKernel
```

A.3 The Paper 06 conditional assembly

```
-- Vol6.YonedaBlaschkeDetectorCalculus
def yoneda_blaschke_detector_calculus_of_subtargets
    (B : BundledSubTargets) :
    YonedaBlaschkeDetectorCalculus
```

A.4 The Vol6 final obstruction

```
-- Vol6.RHClassicalNewLanguage
structure Vol6FinalObstruction : Type 1 where
  bridgeRF01    : OffCriticalDefectKernelBridge
  externalIntro : ExternalizationIntroductionRule
                    burnolBlaschkeCondensedHilbertDefect
  cokernelIntro : CanonicalQuotientOrthogonalityIntroduction
  admissIntro   : CanonicalAdmissibilityIntroduction
```

A.5 The synthesis wrapper

```
-- Vol6.RHClassicalNewLanguage
theorem RH_classical_new_language_of_payload
    (payload : NoPhantomLanguageBreakthrough) :
    RH_classical
```

A.6 The substantive deliverable

```
-- Vol6.RHClassicalNewLanguage
theorem RH_classical_new_language_of_inputs
    (P03 : OffCriticalDefectKernelBridge)
    (P06 : Vol6.BundledSubTargets) :
    RH_classical
```

A.7 The bundled-sub-targets-of-obstruction record

```
-- Vol6.RHClassicalNewLanguage
noncomputable def bundledSubTargets_of_obstruction
    (Ω : Vol6FinalObstruction)
    (S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect)
    (X : DQObj) :
    Vol6.BundledSubTargets
```

A.8 The principal synthesis theorem

```
-- Vol6.RHClassicalNewLanguage
theorem RH_classical_new_language_of_obstruction
    (Ω : Vol6FinalObstruction)
    (S : RKHSModelSpaceDetector burnolBlaschkeCondensedHilbertDefect)
    (X : DQObj) :
    RH_classical
```

B Verbatim #print axioms output

```
'Vol6.RHClassicalNewLanguage.RH_classical_new_language_of_obstruction'
depends on axioms: [propext,
 Classical.choice,
 Quot.sound,
 Vol5.YonedaNymanTrackA.DQHom,
 Vol5.YonedaNymanTrackA.DQ_comp,
 Vol5.YonedaNymanTrackA.HardySubmodule,
 Vol5.YonedaNymanTrackA.NymanKernel,
```

```
Vol5.YonedaNymanTrackA.blaschkeHardyImage,
Vol5.YonedaNymanTrackA.burnolBlaschkeFactor]
```